The Philosophy of *Mathematics in Context*

The design principles for *Mathematics in Context* are derived from Realistic Mathematics Education (RME). Since 1971, researchers at the Freudenthal Institute have developed this theoretical approach towards the learning and teaching of mathematics. RME incorporates views on what mathematics is, how students learn mathematics, and how mathematics should be taught. The principles that underlie this approach are strongly influenced by Hans Freudenthal's concept of "mathematics as a human activity." From the RME perspective, students are seen as reinventors, with teachers guiding and making conscious to students the mathematization of reality. The process of guided reinvention is supported by student engagement in problem solving, a collective as well as an individual activity, in which whole-class discussions centering on conjecture, explanation, and justification play a crucial role. The instructional guidance of teachers in this process is critical, as teachers gradually introduce and negotiate with the students the meanings and use of conventional mathematical terms, signs, and symbols.

The Realistic Mathematics Education philosophy transfers into a design approach for MiC with the following components:

Developing instruction based in experientially real contexts

The starting point of any instructional sequence should involve situations that are experientially real to students so that they can immediately engage in personally meaningful mathematical activity. Such problems often involve everyday life settings or fictitious scenarios, although mathematics itself can also serve as a context of interest. With experience, parts of mathematics become experientially real to students. Such activities should reflect either real phenomena from which mathematics has developed historically or actual situations and phenomena where further interpretation, study, and analysis require the use of mathematics.

Designing structured sets of instructional activities that reflect and work toward important mathematical goals

A second tenet of RME is that the starting point should also be justifiable in terms of the potential end point of a learning sequence. To accomplish this, the domain needs to be well mapped. This involves identifying the key features and resources of the domain that are important for students to find, discover, use, or even invent for themselves, and then relating them via long learning lines. The situations that serve as starting points for a domain are critical and should continue to function as paradigm cases that involve rich imagery and, thus, anchor students' increasingly abstract activity. The students' initially informal mathematical activity should constitute a basis from which they can abstract and construct increasingly sophisticated mathematical conceptions.

Designing opportunities to build connections between content strands through solving problems that reflect these interconnections

The third tenet of RME is based on the observation that real phenomena, in which mathematical structures and concepts manifest themselves, lead to interconnections within and between content strands as well as connections to other disciplines (for example, biological sciences, physics, sociology, and so on). Although the maps developed for each of the four MiC content strands contain unique terms, signs, and symbols as well as an extended learning line for the strand, instruction in actual classrooms inevitably involves the intertwining of these strands. Problems can often be viewed from multiple viewpoints. For example, a geometric pattern can be expressed using numbers relationships or algebraically.

Recognizing and making use of students' prior conceptions and representations (models) to support the development of more formal mathematics (i.e., progressive formalization), RME's fourth tenet is that instructional sequences should involve activities in which students reveal and create models of their informal mathematical activity.

RME's heuristic for laying out long learning lines for students involves a conjecture about the role that emergent models play in the students' learning, namely that students' models of their informal mathematical activity can evolve into models for increasingly abstract mathematical reasoning. Gravemeijer explained this bottom-up progression in terms of four levels of progressive mathematization (see Figure 1).

At the initial "situational level," the expectation is that students develop interpretations, representations, and strategies appropriate for engaging with a particular problem context. At this level, students create and elaborate symbolic models of their informal mathematical activity, such as drawings, diagrams, tables, and informal notations.

At the "referential level," students create informal *models of* the problem situation. Such *models-of* contain the collective descriptions, concepts, procedures, and strategies that refer to concrete or paradigmatic situations. At the "general level," as a result of generalization, exploration, and reflection, students are expected to mathematize their informal modeling activity and begin to focus on interpretations and solutions independent of situation-specific imagery. Models at this level are considered *models-for* and are used as a basis for reasoning and reflection. The "formal level" involves reasoning with conventional symbols and is no longer dependent on *models-for*.

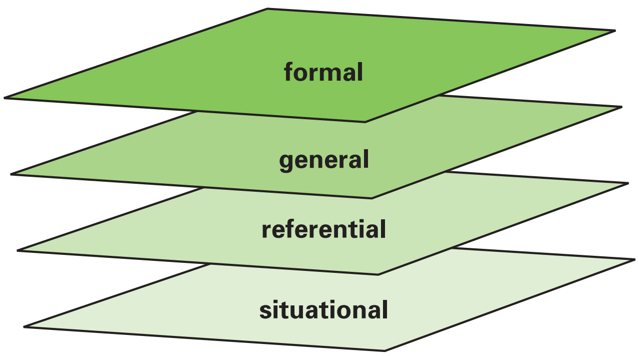

Figure 1.

Thus, for students, a model is first constituted, for example, as a context-specific model of a situation, then generalized across situations. In this process, the model changes in character and becomes an entity in itself, functioning eventually as a basis for mathematical reasoning on a formal level. The development of ways of symbolizing problem situations and the transition from informal to formal notations are important aspects of the selection of problem contexts, the relationships between contexts, and instructional assumptions.

The consequences are that the introduction of numbers, number sentences, standard algorithms, terms, signs and symbols, and the rules of use in formal mathematics should involve a process of *social negotiation* in a manner similar to those from which the notations and rules were derived. In the process of reinventing formal semiotics, students are participating in the human mathematics process of progressive formalization.

The design of activities to promote pedagogical strategies that support students' collective investigation of reality

RME's fifth tenet is that in classrooms the learning process can be effective only when it occurs within the context of interactive instruction. Students are expected to explain and justify their solutions, to work to understand other students' solutions, to agree and disagree openly, to question alternatives, to reflect on what they have discussed and learned, and so on. Creating a classroom that fosters such interaction involves a shift in the teacher's role from dictating the prescribed knowledge via a routine instructional sequence, to orchestrating activities situated in learning trajectories wherein ideas emerge and develop in the social setting of the classroom. To promote the interaction of student representations and strategies, teachers must also create discourse communities that support and encourage student conjectures, modeling, re-modeling, and argumentation.

The development of an assessment system that monitors both group and individual student progress

The teacher's role in the RME instructional process involves capitalizing on students' reasoning and continually introducing and negotiating with students the emergence of shared terms, symbols, rules, and strategies, with an eye to encouraging students to reflect on what they learn. Students are seen as reinventors, with the teacher guiding and making conscious to students the mathematization of reality, with symbolizations emerging and developing meaning in the social situation of the classroom. Students are encouraged to communicate their knowledge, either verbally, in writing, or through some other means like pictures, diagrams, or models to other students and the teacher. Central to this interactive classroom is the development of students' abilities to use mathematical argumentation to support their own conjectures.

A Summary of Implications of Realistic Mathematics Education for MiC

The real-world contexts support and motivate learning. Mathematics is a tool to help students make sense of their world. *Mathematics in Context* uses real-life situations as a starting point for learning; these situations illustrate the variety of ways in which students can use mathematics. Models help students learn mathematics at different levels of abstraction. The ratio tables, double number lines, chance ladders, percent bars, and other models in *Mathematics in Context* allow students to solve problems at different levels of abstraction. The models also serve as mediating tools between the concrete world of real-life problems and the abstract world of mathematical knowledge.

Students reinvent significant mathematics. Instruction builds on students' own knowledge of and experiences with mathematics. They serve as a foundation for developing deeper understanding through interaction with other students and the teacher. The teacher's role is to help students make connections and synthesize what they have learned.

Interaction is essential for learning mathematics. Interaction between teacher and student, student and student, and teacher and teacher is an integral part of creating mathematical knowledge. The problems posed in *Mathematics in Context* provide a natural way for students to interact before, during, and after finding a solution.

Multiple strategies are important. Most problems can be solved with more than one strategy. *Mathematics in Context* recognizes that students come to each unit with prior knowledge and encourages students to solve problems in their own way—by using their own strategies at their own level of sophistication. It is the teacher's responsibility to orchestrate class discussions to reveal the variety of strategies students are using. Students enrich their understanding of mathematics and increase their ability to select appropriate problem-solving strategies by comparing and analyzing their own and other students' strategies.

Students should not move quickly to the abstract. In *Mathematics in Context*, it is preferable that students use informal strategies that they understand rather than formal procedures that they do not understand. It is important to allow students to experience and explore concrete mathematics for as long as they need to. Level 1 units offer students an assortment of informal methods for solving problems. Students are given the opportunity to move to more formal strategies in Levels 2 and 3.

Mastery develops over the course of the curriculum. Because mastery develops over time, teachers should not expect students to master mathematical concepts after a single section or even after a single unit. Each unit is connected to the other units in the curriculum. Through a spiraling of the content and contexts, important mathematical ideas are revisited throughout the curriculum so that students can deepen their understanding and master the ideas over time.

Source: © Rand McNally.

Goals of MiC

The Vision for *Mathematics in Context*

Pedagogically, *Mathematics in Context* is designed to support the National Council of Teachers of Mathematics (NCTM) vision of mathematics education as expressed in the *Principles and Standards for School Mathematics* (NCTM, 2000). It consists of mathematical tasks and questions designed to stimulate mathematical thinking and to promote discussion among students. Students are expected to:

- explore mathematical relationships;
- develop and explain their own reasoning and strategies for solving problems;
- use problem-solving tools appropriately; and
- listen to, understand, and value each other's strategies.

The NCTM *Principles and Standards* state that because mathematical understanding is related to personal experiences in solving problems in the real world, school mathematics is enhanced when it is embedded in realistic contexts that are meaningful to students. *Mathematics in Context* provides these contexts as well as problems that actively engage students in mathematics.

NCTM also suggests that students need ample opportunities to solve realistic problems using strategies that make sense to them. Students must recognize, understand, and extract the mathematical relationships embedded in a broad range of situations. They need to know how to represent quantitative and spatial relationships and how to use the language of mathematics to express these relationships. They must know how and when to use technology. Effective problem-solving also requires the ability to predict and interpret results. *Mathematics in Context* is a connected curriculum that requires students to deepen their understanding of significant mathematics through integrated activities across units and grades.

The History of *Mathematics in Context*

The *Mathematics in Context* curriculum project was initially funded in 1991 by the National Science Foundation to develop a comprehensive mathematics curriculum for the middle grades. Collaborating on this project were the research and development teams from the National Center for Research in Mathematical Sciences Education (NCRSME) at the University of Wisconsin, Madison, and the Freudenthal Institute (FI) at the University of Utrecht in the Netherlands. Thomas A. Romberg, Project Director, had become familiar with the work of the Freudenthal Institute while chairing the writing team of the NCTM's *Curriculum and Evaluation Standards* (1989). He recognized that the mathematics curriculum used in the Netherlands was consistent with the vision of the Standards and that it could serve as a model for a middle grades curriculum for the United States.

The development of the curriculum units, teacher's guides, and supporting materials took six years (1991–1997). An international advisory committee prepared a blueprint document to guide the development of the curricular materials. Then the Freudenthal Institute staff under the direction of Jan de Lange prepared initial drafts of individual units based upon the blueprint. Researchers at the University of Wisconsin–Madison modified the language and problem contexts in these units to make them appropriate for students and teachers in the United States. Pilot versions of the individual units were tested in middle schools in Wisconsin. Revised field-test versions of the units were created from feedback received during the initial pilot. The revised versions were then used in several states and Puerto Rico. Data from the field-test sites were used to inform revisions to student books and teacher's guides before commercial publication.

The current revision of *Mathematics in Context* (2003–2005) was partially funded by the National Science Foundation. The revision team solicited feedback from experienced MiC teachers. This information was used in conjunction with the NCTM 2000 *Principles and Standards* to shape the changes reflected in the 2006 edition. The staff of the Freudenthal Institute prepared drafts of each revised unit similar to the first edition. The revision team at the University of Wisconsin–Madison then prepared the drafts for field-test, if necessary, and for submission to the publisher. Extensive teacher's guide notes were contributed by experienced MiC teachers.