Overview for Families

Mathematics in Context unit: ***Ups and Downs***

Mathematical strand: **Algebra**

The following pages will help you to understand the mathematics that your child is currently studying as well as the type of problems (s)he will solve in this unit.

Each page is divided into three parts:

- *Section Focus*
  Identifies the mathematical content of each section.

- *Learning Lines*
  Describes the mathematical flow of each section.

- *Learning Outcomes*
  Outlines what students should know and be able to do at the end of each section.

"From the very beginning of his education, the child should experience the joy of discovery."

Alfred North Whitehead

Ups and Downs

Section A Trendy Graphs

Section Focus

Students investigate growth over time within a variety of contexts using statements (stories), tables, and graphs. They create and use line graphs to represent continuous processes that occur over time. Students describe growth patterns in their own words, using expressions such as "faster and faster" or "increased every week by equal amounts." Lines are used to connect data so that a trend becomes visible. The formal term *linear growth* is used. The instructional focus of Section A is to

• identify and describe growth patterns from a story, table, or graph;

• create line graphs from data in a table; and

• solve problems involving growth patterns.

Learning Lines

Discern Different Types of Graphs

This section focuses on growth patterns, shown in a variety of graphs:

• a diagram of the thickness of rings of a tree;

• a line graph representing a child's growth over a number of years;

• growth curves representing average growth of a child over a number of months;

• a straight line representing linear growth; and

• curved lines, showing a growth pattern that becomes "faster and faster" or "less and less."

Different Representations for a Relationship

Students use tables to identify patterns. They interpret growth patterns by looking at the graph and are introduced to the concept of different rates of increase (or decrease). By relating growth patterns to graphical representations, students recognize the faster the growth, the steeper the graph. They use the tables and graphs to predict future growth and make connections between situation, graph, and table.

Learning Outcomes

Students are able to write a story to describe growth patterns in a table or graph, and recognize the power of graphs and/or tables for representing and solving problems. Students begin to identify and describe patterns of increase or decrease and reason about the situations in which they occur.

Ups and Downs

Section B Linear Patterns

Section Focus

The situations studied in this section all involve equal increase over equal time periods or a constant rate of growth. This plays an important role in making formulas to represent these situations. For students, it is not obvious that all situations investigated in this section can be described by the same mathematical concept of linear growth. It is the focus of this section to recognize and use linear growth by looking at the differences in a table; looking at the graph, which is a straight line; or looking at the formula that describes the situation. Both recursive (NEXT-CURRENT) and direct formulas are used.

Learning Lines

Linear Relationships

Students investigate linear growth in a variety of situations. They identify whether or not the growth is linear and use tables, graphs, and formulas to describe linear growth. Direct formulas, such as $L = 2 + 1.5 \times T$, are used and interpreted. The first differences in a table of values for a linear formula are equal for equal periods of time.

Students solve problems by comparing graphs and formulas involving linear relationships within the context of comparing two motorcycle rental companies. They informally identify steepness of the graph as represented by the price per mile and the y-intercept as the fixed starting amount. The point of intercept is only introduced informally by comparing graphs that represent both companies' formulas for calculating rental prices.

Learning Outcomes

Students write and use recursive and direct formulas to represent linear growth patterns. They solve problems by comparing different linear growth patterns. They know how to recognize linear growth within a variety of contexts by considering differences in a table or by considering the shape of a graph.

Ups and Downs

Section C Differences in Growth

Section Focus

The surface area of a poplar leaf is used to investigate quadratic growth. Students investigate second differences in a table. They use quadratic formulas that describe quadratic growth within this context. The growth of aquatic weeds is used to introduce students to exponential growth.

Learning lines

Quadratic Relationships

If the "first differences" are not equal but the "second differences" are, the growth is *quadratic*. The table shows the relationship between length and area of squares. If the length increases by one centimeter each time, the first differences are not equal, but the second differences are equal. At this stage, students are not expected to construct quadratic formulas, but they use and adapt them. The formal term *parabola*, for the graph of a quadratic relationship, is not used.

Relationships Showing Exponential Growth

Students are introduced to situations involving exponential growth. In the table, this becomes visible if, at equal time periods, each entry in the table is found by multiplying by a growth factor. If each value in the table is found by multiplying by a growth factor, the growth is *exponential*. Only NEXT-CURRENT formulas are used to describe exponential growth.

Learning Outcomes

Students are able to discern whether growth as presented in a situation using tables is linear, quadratic, exponential, or none of these. They use and understand direct formulas for quadratic growth and recursive formulas for exponential growth.

Ups and Downs

Section D Cycles

Section Focus

Section D introduces the terms *cycle* and *period* to describe features of a periodic phenomenon. Periodic graphs are investigated within the context of tides, temperature changes in an air-conditioned room, blood pressure, and laps on a racetrack.

Learning Lines

Discern Different Types of Graphs

In Section A, students investigated a variety of graphs. In this section, a new type of graph is introduced, the periodic graph. Students interpret and draw a tidal graph that represents the change in water level in tidal regions. They describe the increase and decrease of the temperature in an air-conditioned room and identify the period and cycle of several periodic graphs. They do not use formulas and/or tables that describe periodic phenomena.

Learning Outcomes

Students identify the characteristics of a periodic graph and are able to identify the cycle and period in a periodic graph.

Ups and Downs

Section E Half and Half Again

Section Focus

Students already investigated exponential growth in Section C. Each new value was determined by multiplying by a growth factor. In this section, growth factors between zero and one are used to describe exponential decay in real-life situations like decrease of the value of a car over time and the absorption of some kind of medicine in the blood stream. The relationship between percents and fractions is applied.

Learning Lines

Relationships Showing Exponential Growth
Exponential decay is sometimes referred to as *negative growth*. The growth factor is not negative but a decimal or fraction between 0 and 1.

Number Sense
While working through this section, students show their understanding of the relationship between fractions, decimals, and percents. For example, to calculate one-half of the medicine left in the blood stream after ten minutes, students may either multiply the amount by 1/2, find 50% of the amount, or divide the amount by 2 since these expressions are equivalent to each other.

Learning Outcomes

Students use decimal and fraction growth factors to represent exponential decreasing patterns. They interpret and use tables, graphs, and NEXT-CURRENT formulas to describe exponential decay.