Overview for Families

Mathematics in Context unit: ***Triangles and Beyond***

Mathematical strand: **Geometry and Measurement**

The following pages will help you to understand the mathematics that your child is currently studying as well as the type of problems (s)he will solve in this unit.

Each page is divided into three parts:

- *Section Focus*
  Identifies the mathematical content of each section.

- *Learning Lines*
  Describes the mathematical flow of each section.

- *Learning Outcomes*
  Outlines what students should know and be able to do at the end of each section.

"From the very beginning of his education, the child should experience the joy of discovery."

Alfred North Whitehead

Triangles and Beyond

Section A Triangles and Parallel Lines

Section Focus

In this section, students start to investigate the two topics that play an important role in the entire unit: triangles and (families of) parallel lines. Students identify triangles in two- and three-dimensional situations in pictures and in the world around them. They investigate and describe the differences between lines that are parallel and lines that are not parallel. Three properties of parallel lines are formalized. At the end of this section, students identify families of parallel lines and investigate the triangular grid that families of parallel lines can make. The instructional focus of Section A is to

• identify triangles in various diagrams and pictures;

• investigate how perspective affects the shape of a triangle; and

• solve problems involving parallel lines.

Learning Lines

Parallel Lines

This section introduces the properties of parallel lines:

• They are always the same distance apart.

• They never touch.

• They form equal angles with a line that intersects them.

(The name for this intersecting line is *transversal*; students do not have to know this name here.) Students will use these properties in Section F, where they investigate parallelograms.

Families of Parallel Lines

In Section C, families of parallel lines are used to "proof" the property of the angles of a triangle: The sum of the angles is 180°. In the unit *It's All the Same,* students investigate the properties of congruent and similar triangles, using a triangular grid that is made by three families of parallel lines.

Representation, Spatial Abilities

Students explore three dimensional drawings; they draw side views and front views; and they will be aware that actual shapes of objects cannot always be seen in a three-dimensional drawing because of the perspective.

Learning Outcomes

Students have further developed their spatial abilities. They have developed their understanding of the concept of parallel lines and families of parallel lines. They know properties of parallel lines. They can find equal angles when a line intersects two parallel lines.

Triangles and Beyond

Section B The Sides

Section Focus

Students try to make triangles with various sets of three sticks of different lengths and discover a generalization: the sum of the lengths of any two sides must be greater than the length of the third side. Students start to classify triangles according their side lengths: equilateral, isosceles, and scalene triangles. At the end of the section, students learn to construct triangles given side length using a pair of compasses. The instructional focus of Section B is to

• investigate and apply the geometric property that the two shorter sides of a triangle are always longer than the longest side;

• identify equilateral, isosceles, and scalene triangles; and

• construct a triangle given the length of its sides.

Learning Lines

Properties of Triangles

When students have tried to construct triangles given side lengths, they described in their own words the requirements for the side length. This is described more formally in the Summary. For a triangle, the sum of the lengths of any two sides is greater than the length of the remaining side.

Classification of Triangles

In this section, students classify triangles according their side lengths (equilateral, isosceles, and scalene triangles). In Section D, students classify triangles according the measurements of the angles.

Constructions

To draw a point at a certain distance from point *A* can be done using a centimeter ruler. Students will review that a pair of compasses is the best tool to use when they have to find all locations that are at a certain distance from point *A*. This understanding is essential in order to be able to construct a triangle given side lengths.

Learning Outcomes

Students can recognize and classify equilateral, isosceles, and scalene triangles. They can explain whether or not a triangle can be constructed using the given side lengths. They start to learn how to use a pair of compasses for constructions.

Triangles and Beyond

Section C Angles and Triangles

Section Focus

Students explore the angles of a triangle. They cut out three angles that are 180º together to create a triangle, and they discover that three angle pieces whose measures total more than 180° will not make a triangle. They also see that cutting of the three angles of a triangle will produce angle pieces that form a semicircle. Students investigate the properties of isosceles and equilateral triangles with respect to the angle measurements. At the end of the section, they apply what they have learned to calculate angles of triangles. The instructional focus of Section C is to

• investigate and apply the geometric property that the sum of the angles in a triangle is 180°;

• construct a triangle from two angles and one side; and

• investigate and apply the geometric property that the longest (shortest) side of a triangle is opposite the largest (smallest) angle.

Learning Lines

Properties, Similar and Congruent Triangles

When students create a triangle from three angle pieces whose measures total 180°, they will notice that they can make smaller and larger triangles using the same three pieces. This activity is also related to the similarity postulate AAA (Angle-Angle-Angle): For any two triangles, if the corresponding angles are congruent, then the triangles are similar. Note that the activity in Section B with the pieces of uncooked spaghetti is related to the SSS (Side-Side-Side) postulate: Two triangles are congruent if three sides of one triangle are congruent to three sides of the other triangle. The development of the concept of congruency starts in Section E. Similarity and congruency are investigated more formally in the unit *It's All the Same*.

Proofs

The sum of the three angles of a triangle is 180°. At an informal level, students explored this property in the unit *Looking at an Angle*. In this unit, there are different ways to show that cutting the three angles of a triangle forms a semicircle. This is not a formal proof, but for students, it is a useful way to demonstrate the plausibility of this property. A more formal proof is based on one of the properties of parallel lines and equal angles.

Notation

The symbol $\triangle$ for the word triangle is introduced.

Learning Outcomes

Students know and can use the properties of the sides and angles of isosceles and equilateral triangles. Students know and can use the rule that the sum of the angle measurements in a triangle is 180°.

Triangles and Beyond

Section D Sides and Angles

Section Focus

Students investigate the generalization: The longest side of a triangle is opposite the largest angle. They classify triangles according to the size of the largest angle: acute, right, and obtuse triangles. Students are introduced to the Pythagorean theorem. By studying the area of squares that fit on the sides of triangles, they will discover that for a right triangle there is a special relationship: If a triangle has a right angle, then the square on the longest side has the same area as the other two squares combined.

Learning Lines

Properties of Triangles
The generalization "The longest side of a triangle is opposite the largest angle" is known as the Opposite Parts theorem or Hinge theorem, but students do not have to know these names.

Measurement, Area, and Length
Students calculate the area of squares drawn on graph paper. They use strategies from the unit *Reallotment* to calculate the area of the squares. Students use the area of a square to find the side length of the square by "unsquaring" or taking the square root of the area. The idea of unsquaring and the square root symbol $\sqrt{}$ was introduced in the unit *Building Formulas*.

The Pythagorean theorem states that in a right triangle, the square of the hypotenuse is equal to the sum of the squares of its legs. In this section, students are informally introduced to the Pythagorean theorem. This means that this theory is described as a relationship of the area of the squares that fit on the sides of a right triangle. This is done to give students visual support and to help them to make their calculations correctly.

Notation
The angle symbol to designate right angles is introduced in this section.

Learning Outcomes

Students can recognize and classify acute, right, and obtuse triangles. They are able to use the relationship between the area and the side length of a square. They understand and use the Pythagorean theorem.

Triangles and Beyond

Section E Congruent Triangles

Section Focus

Students learn how shapes can be flipped, turned, or slid to create a desired effect. In this way, they are introduced to the transformations reflection, rotation, and translation. They are introduced to the concept of congruent figures. They start to investigate the properties of the corresponding side length and angle measurements of two congruent figures. Students look for symmetry in shapes and for lines of symmetry. The instructional focus of Section E is to

- **solve problems involving translations, rotations, and reflections;**
- **identify congruent figures; and**
- **identify a line of symmetry.**

Learning Lines

Concepts: Congruency and Transformations

The context of making copies using stamps and stencils is used to introduce the concept of congruency. At this stage, students start to work with an informal description of the concept of congruent figures: figures that are copies of each other. Congruent figures have the same size and shape; they can fit on top of each other exactly with no overlap.

In order to see if two figures are congruent, one of the figures can be moved around until it fits on the other one. Students learn to describe these movements (transformations) using formal language: translation, rotation, and reflection. In Section F, translations are combined to construct regular polygons, parallelograms, and rectangles.

Learning Outcomes

Students can solve problems involving translations, rotations, and reflections. They can identify congruent figures. They understand and can identify line symmetry and line of reflection.

Triangles and Beyond

Section F Triangles and Beyond

Section Focus

Students construct parallel lines and families of parallel lines using a straightedge and a plastic triangle. They construct parallelograms using strips and learn to identify different types of parallelograms (i.e., rectangle, rhombus, and square). Students construct regular polygons, parallelograms, and rectangles by rotating and translating triangles. The instructional focus of Section F is to

- **draw parallel lines using a straightedge and a cardboard triangle;**
- **solve problems involving parallel lines;**
- **identify and describe parallelograms, rectangles, rhombi, and squares; and**
- **describe the relationship between polygons and their central angles.**

Learning Lines

In this last section, many topics studied in previous sections return and are combined.

Properties: Parallelograms

Students start to explore the properties of special parallelograms: A rectangle is a parallelogram with four equal angles that each measure 90°. A rhombus is a parallelogram with four equal sides, but not necessarily four equal angles. A square is a parallelogram with four equal sides and four equal angles. The characteristics of parallelograms are emphasized when they are constructed from a triangle and its image. This image is the result of a rotation of the original triangle around one of the vertices, followed by a translation.

Properties: Polygons

Polygons were introduced in the unit *Packages and Polygons*. In this section, students make regular polygons by rotating a triangle with the correct vertex angle, which must be a factor of 360. (The concept of a factor is introduced in the unit *Facts and Factors*.) For example, 60 is a factor of 360. You can trace a triangle with a vertex angle of 60° and rotate it five times to create a hexagon.

Learning Outcomes

Students are able to construct parallel lines and families of parallel lines. Students can recognize and classify quadrilaterals (i.e., parallelogram, rectangle, rhombus, and square). Students explore a way to construct regular polygons. Students solve problems involving translations, rotations, and reflections.