Overview for Families

Mathematics in Context unit: ***Take a Chance***

Mathematical strand: **Data Analysis and Probability**

The following pages will help you to understand the mathematics that your child is currently studying as well as the type of problems (s)he will solve in this unit.

Each page is divided into three parts:

- *Section Focus*
  Identifies the mathematical content of each section.

- *Learning Lines*
  Describes the mathematical flow of each section.

- *Learning Outcomes*
  Outlines what students should know and be able to do at the end of each section.

"From the very beginning of his education, the child should experience the joy of discovery."

Alfred North Whitehead

Take a Chance

Section A Fair

Section Focus

Students evaluate the fairness of decisions made in a number of real-life contexts. They explore theoretical and experimental chance, using number cubes, spinners, coins, and other objects. The instructional focus of Section A is to

- **review and apply the terms *fair* and *chance*;**
- **conduct simple probability experiments; and**
- **explore a variety of tools for making a fair decision.**

Learning Lines

Concepts *Chance* and *Fair*
This section is on fairness; the concept of chance is always related to fair in this section. The term *chance* is used explicitly only a few times in this section. In the beginning of the unit, students are asked to describe what they think the word *chance* means. Later, for instance when students describe what is meant by *fair* or when students describe how they can determine whether a particular method for making decisions will be fair, students may spontaneously use the word *chance*. The word *chance* is explicitly used in the context of tossing coins. Here an "informally" numerical value is connected to chance. It is stated that "there's a 50-50 chance of getting heads or tails." In Section A, except for this example, no numerical values are connected to chance. In the Summary, the word *chance* is used in the definition of fair.

Making Fair Decisions
Students explore if and how coins, number cubes, spinners, and irregular objects like paper cups can be used to make fair decisions. They informally deal with *theoretical chance* when reasoning about the fairness of different methods to make decisions, like rolling number cubes, tossing coins, and spinning spinners.

This is a typical example of such a problem:

> Hillary and Robert have a black-and-white cube. Hillary wins if it comes up white, and Robert wins if it comes up black. How can you tell if the cube has been colored in a fair way?

For this type of problem, students do not need to do experiments; they can find the answers by reasoning about chances. Doing experiments with "chance objects" and recording the results is a preparation for dealing with *experimental chance*. The experiments are done in this section, but chance is connected to the outcomes later in Section C. At that point, students have seen ways to calculate chance and express it as a number.

Models
The tree diagram is introduced as a visual model to show all possible outcomes. In the rest of the unit, this model will be used to count all possible and favorable outcomes.

Learning Outcomes

Students are able to decide how to make fair decisions in given situations in different ways. Students can judge whether methods to make decisions are fair.

Take a Chance

Section B What's the Chance?

Section Focus

The instructional focus of Section B is to

- **use visual models to estimate and calculate chance and**
- **calculate and express chance for simple situations using percents, fractions, or ratios.**

Learning Lines

Chance as Having a Numerical Value

In this section, students gain a basic understanding of the concept of chance by estimating chances and describing them using qualitative descriptions such as "sure" and "not sure," before describing exact chances using percents, fractions, or ratios.

Students estimate and express chance first in terms like "sure to happen," "sure not to happen," and "not sure." They order chances along this dimension using a visual model, the chance ladder.

When ordering chances within the category "not sure," students need to find a way to compare the chances and refine this category. This will help students develop the understanding that chances can be expressed with a number.

Ordering chances that can be calculated will help students find the numerical value of a chance. Students do not need to master "calculating chances" at this stage. This will be dealt with repeatedly later in this unit as well as in the following unit in this strand, *Second Chance*.

Ways to Express Chance

In this section, chance is first expressed in everyday qualitative language like "It is sure to happen," "It probably will," and "There's a 50-50 chance." Next, chance is expressed quantitatively as a percentage, by placing chances on a ladder with a scale ranging from 0% to 100%. This is done only for simple percentages, like 50% and 25%, and by ordering.

Chance is also expressed as a ratio "so many out of so many." This ratio is connected to the corresponding fraction. A connection is made to Section A by having students order chances for spinners. Here students cannot use the ratio notation based on counting. They can, however, use fractions and connect ratios and fractions. Students need to have some basic ability in working with benchmark fractions, percents, and ratios and relate these to one another.

Models

The chance ladder is introduced as a visual model to express and order chances on a scale from "sure not to happen" to "sure to happen," with nine rungs in between. On the same ladder, later a scale ranging from 0% to 100% is added to the qualitative scale. Then each rung can be assigned a value as a percentage in multiples of 10%. The ladder is then modeled into a scale line ranging from 0% to 100%. Students are also asked to put fractions on the scale. The model is slowly evolving from a model of a situation to a model for ordering chances. The scale line, which uses only the percents, is like a number line.

Learning Outcomes

Students are able to order chances and express chance using ratios, fractions, or percents.

Take a Chance

Section C Let the Good Times Roll

Section Focus

Students predict results, record outcomes, and analyze the results of simple chance experiments. Students develop an informal understanding of the difference between theoretical and experimental chance. The instructional focus of Section C is to

- **estimate chance from repeated trials of an experiment and**
- **explore theoretical and experimental probability.**

Learning Lines

Experimental and Theoretical Chance
The difference between theoretical and experimental chance is informally addressed. What students expect or predict to happen (theoretical chance) is compared to what actually did happen in the experiment of rolling a number cube (experimental chance). Students experience that by conducting repeated trials of an experiment, like rolling number cubes or tossing a coin, the outcomes will approach the expected outcomes.

Students also realize that when tossing a coin or rolling a number cube, the next outcome is not affected by the previous outcomes. Students estimate or calculate experimental chances as "so many out of so many."

Students do not need to know these terms. In the unit *Second Chance*, more attention is paid to the subject of experimental and theoretical chances.

Variability
By conducting experiments and studying outcomes, students experience that variability will occur in the outcomes. Problem 7 on page 21 is a good problem to discuss this issue.

Models
Students use different types of tables to record results of experiments. Frequency tables as well as cumulative frequency tables are used. The cumulative frequency table can show both the variability of outcomes as well as the fact that the experimental chance approaches the theoretical chance if the number of trials gets bigger. This type of table is used for this purpose in the unit *Second Chance*.

Mathematical Background to Experimental and Theoretical Chance
Theoretical chance is based on the possible outcomes in a particular experiment and can be determined before the experiment is conducted. Calculating theoretical chance is straightforward when each outcome has an equal chance of occurring. For number cubes, coins, and simple spinners, the theoretical chance can be calculated by dividing the number of selected outcomes by the total number of outcomes. This way of calculating theoretical chances is addressed in the next section. Experimental chance is based on the trials of an experiment and is calculated by dividing the number of times an outcome occurs by the total number of trials. For a large number of trials, experimental chance approximates theoretical chance.

When rolling number cubes and tossing coins, the theoretical chance of each outcome is known. When tossing or rolling irregularly shaped objects, this is not the case. The only way to get insight into the chances for the different outcomes is through repeated trials.

Take a Chance

Section C Let the Good Times Roll

Learning Outcomes

Students can estimate chance from repeated trials of an experiment. They know that the experimental chance can be different from what they predicted, but that in the long run for a chance experiment, results will get closer and closer to what they expected.

They also know informally that (for independent events) an outcome is not affected by previous outcomes; for example, tossing ten heads in a row does not affect the chance that the next coin toss will be a head. The chance remains 50%.

Take a Chance

Section D Let Me Count the Ways

Section Focus

Students explore chance situations involving multiple events and determine possible outcomes using tree diagrams and tables. The instructional focus of Section D is to

- **determine all combinations and possible outcomes using tree diagrams and tables;**
- **simulate simple events by using coins and number cubes to estimate or check the chances for possible outcomes; and**
- **calculate chance for simple situations involving multiple events by counting possibilities.**

Learning Lines

Possible Outcomes

In this section, students use a variety of methods to represent and count the possible outcomes in different multi-event situations. At first students can use their own way to show all possible outcomes. They can, for example, use drawings, lists, diagrams. Later the tree diagram and a "table" are introduced as a way to do this.

Chance in Multi-Event Situations

Students first predict the theoretical chance in a multi-event situation, represented in a tree diagram. The context is the possible combinations of boys and girls in families with two or more children. Students use a simulation with coins to find the experimental chance.

After finding all possible outcomes, students can find the chances on any outcome as the ratio of the number of favorable outcomes out of all the possible outcomes. This idea was addressed informally in Section B in the context of the tiled floor. Here it is revisited in a pre-formal way on page 30. A formal chance definition based on this way of finding chances is introduced in the unit *Second Chance*.

The idea of complementary chances is informally addressed. Here, for example, students are asked to find the chance of "not rolling a sum of 10 with two number cubes." This is revisited in the unit *Second Chance*.

Models

The tree diagram, which was introduced in Section B, is extended to represent combined events, like choosing a T-shirt and choosing a pair of pants or having a first child and having a second child or the way Hillary holds her hand and the way Robert holds his. Students use tree diagrams to represent and determine the likelihood of possible permutations. By tracing the paths along the branches, students can find all favorable and all possible outcomes. A grid or table is also used to show and count all possible outcomes and find chances.

Learning Outcomes

Students can list and count all possible outcomes in multi-event situations involving two or three combined events by using tree diagrams or other methods. They can determine the chance a single outcome will occur by finding the ratio of the number of favorable outcomes to all possible outcomes.