Overview for Families

Mathematics in Context unit: ***Second Chance***

Mathematical strand: **Data Analysis and Probability**

The following pages will help you to understand the mathematics that your child is currently studying as well as the type of problems (s)he will solve in this unit.

Each page is divided into three parts:

- *Section Focus*
  Identifies the mathematical content of each section.

- *Learning Lines*
  Describes the mathematical flow of each section.

- *Learning Outcomes*
  Outlines what students should know and be able to do at the end of each section.

"From the very beginning of his education, the child should experience the joy of discovery."

Alfred North Whitehead

Second Chance

Section A Make a Choice

Section Focus

Students investigate situations in which several choices need to be combined like choosing pants, a T-shirt, and shoes for a possible outfit. They use tree diagrams, tables, ordered lists, and smart calculations to find the total number of possible outcomes. They also reason about the theoretical chances for outcomes, which can be found by applying the following rule:

$$\text{Chance} = \frac{\text{Number of favorable outcomes}}{\text{Total number of possible outcomes}}$$

Learning Lines

Theoretical Chance

In this section, a formal rule to find theoretical chances on outcomes is presented. Chance is defined as:

$$\frac{\text{Number of favorable outcomes}}{\text{Total number of possible outcomes}}$$

Students use this rule in several situations, such as in the situation of finding the chance that a three-child family will have three boys. (Note: the assumption is made that the chance of having a boy or a girl are equal, which is not quite true if you study the statistics on births.) Chances are written as ratios, fractions, or percents and sometimes as decimals. Decimals are mostly only used as an in between result when converting ratios or fractions to percents.

Models

The tree diagram that was introduced in the unit *Take a Chance* is used in this section to model and structure several situations involving combined events (like choosing an outfit or rolling two number cubes) or a sequence of events (like having three children). Students use this model to count possible and favorable outcomes. Later in the unit, in Section D, the tree diagram is transformed into a chance tree. In addition to the tree diagram, a table is used as a model to structure and list all possible outcomes. This is done to model the outcomes for rolling two number cubes, since a tree diagram would have become 36 endpoints and be rather cumbersome.

Learning Outcomes

To find chances, students can use the rule:

$$\text{Chance} = \frac{\text{Number of favorable outcomes}}{\text{Total number of possible outcomes}}$$

They are able to use several models to structure and model situations involving different outcomes.

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Section B A Matter of Information

Section Focus

Students investigate information collected through surveys, sampling, and experiments. They use relative frequencies of outcomes to make statements about experimental (or empirical) probabilities. These statements are only relevant to future events if the situations are repeatable and do not influence each other. In other words, the events should be independent, and the chance for each outcome should be the same for each future event. With experiments, many trials are often required before the experimental probability approaches the theoretical probability.

Learning Lines

Experimental Chance

In this section, experimental (or empirical) chance is explored. Students investigate several situations in which information is collected to estimate the frequency of the possible outcomes. This information can be presented in frequency tables, bar graphs, or other diagrams. The ratio used to find experimental chance is comparable to the rule for theoretical chance. The chance of a favorable outcome occurring is

$$\frac{\text{Number of Favorable Outcomes}}{\text{Number of All Possible Outcomes}}$$

Students collect data by recording the frequency of letters in a sample English text. They use their data to make statements about the experimental chance of randomly selecting certain letters and compare the results of their own sample to relative frequencies of letters from large-scale sampling.

Related Events

Chances depend on what you know. Two-way tables can be used to determine if two events are dependent. This notion is pre-formally addressed in this section and formalized in the next probability unit *Great Predictions*.

Learning Outcomes

Students know the difference between theoretical and experimental probability. They can complete two-way tables and make chance statements that fit this information. From information in two-way tables, students can decide whether events are "related" (or dependent).

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Section C In the Long Run

Section Focus

Students investigate what happens to the chances (relative frequencies) on the outcomes as the number of trials increases. They note that the variation in the results becomes smaller, the results become more stable, and the experimental chance approaches the theoretical chance. Students use experiments and simulations and calculate experimental chances in several situations.

Learning Lines

Games of Chance

Chance games often involve situations with combined events. Before playing such games, it is useful to determine each player's chance for winning to determine whether the game is fair or unfair. A game's fairness depends on each player's chance of winning and on the scoring system.

Learning Outcomes

Students recognize that in the long run experimental probabilities approach theoretical probabilities. They are able to find the theoretical chance for a situation or estimate the experimental chance. They make informed decisions about whether games are fair and investigate whether certain strategies offer a better chance of winning a game. Students know how to set up and conduct a simulation if theoretical chances are difficult to calculate.

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Section D Computing Chances

Section Focus

Students revisit some of the situations from Section C. They learn how to compute the theoretical chances for these situations using a chance tree and the multiplication of chances. The area model is introduced as another model that can be used to find chances for a situation with two combined events.

Learning Lines

Theoretical Chance

In this section, students revisit several situations with combined events from Section C. In Section C, students used experiments and simulations to find experimental probabilities. In this section, they learn how to calculate the theoretical chances for combined events. Several concepts are reinforced, including the inherent variability in the results of an experiment and the need for many trials for experimental chances to approach theoretical chances.

Models and Tools

The tree diagram is converted into a chance tree. With a chance tree, it is not possible to count the endpoints to find all possible outcomes. Favorable outcomes cannot be "counted" either. However, routes for favorable outcomes can still be followed through the diagram to compute probabilities. Each route is labeled with the chance for a given outcome. Instead of counting routes and using the ratio of favorable outcomes to possible outcomes, chances are directly found by multiplying the chances along the route. The multiplication rule for chance is further formalized in the unit *Great Predictions*.

The area model is also introduced in this section as another way to find chances for two combined or successive events. Using an area model, students divide a rectangle or square in two directions to represent the number of outcomes for each event. The area of the whole rectangle represents a chance of 1, while the areas of the (un)shaded parts represent chances for the different outcomes as well as combined outcomes. Students can find chances as a ratio of the number of small units for a given part divided by the total number of small units in the rectangle. They can also multiply the lengths and widths (fractions) representing chances for each outcome.

Expected Value

The idea of expected value is explored further using the game of Hog. Expected value is not yet formalized; however, students focus on the expected score to determine which group of number cubes might produce the best score.

Learning Outcomes

Students can make a chance tree or an area model to represent a combined event situation. They can also calculate chances for combined events using the multiplication rule. Students can express these chances as fractions, ratios, decimals, or percents.