Overview for Families

Mathematics in Context unit: ***Revisiting Numbers***

Mathematical strand: **Number**

The following pages will help you to understand the mathematics that your child is currently studying as well as the type of problems (s)he will solve in this unit.

Each page is divided into three parts:

- *Section Focus*
  Identifies the mathematical content of each section.

- *Learning Lines*
  Describes the mathematical flow of each section.

- *Learning Outcomes*
  Outlines what students should know and be able to do at the end of each section.

"From the very beginning of his education, the child should experience the joy of discovery."

Alfred North Whitehead

Revisiting Numbers

Section A Speed

Section Focus

The focus of this section is to develop an understanding of ratios and rates, to introduce the concept of rates formally, and to review and further develop students' understanding of the exponential notation and the scientific notation.

Learning Lines

Ratios, Averages, and Rates

In the unit *Ratios and Rates,* students used ratios to find averages such as average speed (mi/h) or gas mileage (mpg). An average can be considered as a ratio in which something is compared to one unit (or a rate). The average, as mean, was introduced in the units *Picturing Numbers* and *Dealing with Data*. In the unit *Ratios and Rates,* students used ratio tables and calculators to write ratios as single numbers. In Section A of *Revisiting Numbers,* students review and further explore rates, especially speed rate. They will also solve more complicated problems when they have to calculate, for example, the speed in kilometers per hour to a given distance in meters and a time in seconds.

Learning Outcomes

Students have further developed number sense and a conceptual understanding of rates. They have experiences in solving problems about speed rate and units change (e.g., from km/h into m/s). They deepened their understanding of scientific notation, and they reflected on operations with powers of ten at a more formal level.

Revisiting Numbers

Section B Notation

Section Focus

The focus of this section is to develop an understanding of the base-ten number system, to review and further develop the notation of large and small numbers using positive and negative powers of ten, and to introduce and reinforce students' understanding of the scientific notation for small and large numbers.

Learning Lines

Number Systems and Number Sense

Students will use and further develop their number sense and computational skills when they study place value and decimal numbers. When multiplying a number by ten, the value of each digit changes to a higher value. When dividing a number by ten, the value of each digit changes to a lower value. For example, $14.75 \div 10 = 1.475$.

Operations

In this section, two rules for operations with powers of ten are investigated and formalized: If m and n are whole numbers, then $10^m \times 10^n = 10^{m+n}$ and $10^m \div 10^n = 10^{m-n}$.

Exponential Notation

The context of dilution is used to introduce negative powers of ten. In each dilution, the amount of food coloring is divided by ten. Arrow language can be used to describe the dilution process.

$$1{,}000 \xrightarrow{\div 10} 100 \xrightarrow{\div 10} 10 \xrightarrow{\div 10} 1 \xrightarrow{\div 10} 0.1 \xrightarrow{\div 10} 0.01 \xrightarrow{\div 10} 0.001$$

When the numbers are written as powers of ten, a pattern in the exponents is visible, which naturally leads to negative exponents:

$$10^3 \xrightarrow{\div 10} 10^2 \xrightarrow{\div 10} 10^1 \xrightarrow{\div 10} 10^0 \xrightarrow{\div 10} 10^{-1} \xrightarrow{\div 10} 10^{-2} \xrightarrow{\div 10} 10^{-3}$$

The study of negative powers of ten is a base to develop students' ability to read a calculator display that shows a negative power of ten.

Scientific Notation

In Section A, scientific notation is reviewed from the unit *Facts and Factors,* using positive powers of ten. Section B introduces negative exponents. In general, a number written in scientific notation is a product of a number between 1 and 10 and a power of ten.

$45{,}000{,}000 = 4.5 \times 10{,}000{,}000$, so 45,000,000 written in scientific notation is 4.5×10^7.
$0.000025 = 2.5 \times 0.00001$, so 0.000025 written in scientific notation is 2.5×10^{-5}.

Revisiting Numbers

Section B Notation

Learning Outcomes

Students have further developed their number sense and a conceptual understanding of the base-ten number system and powers of ten with positive and negative exponents. They are able to interpret a large number shown on a calculator as a product of a number and a positive or a negative power of ten and write this product as a single number. They begin to generalize operations with powers of ten.

Revisiting Numbers

Section C Investigating Algorithms

Section Focus

In this section, students put themselves in the role of a math tutor for a fourth-grade boy. Students compare and contrast various strategies for multiplying two- and three-digit numbers. These strategies include using ratio tables, finding partial products, adding to calculate the final product, using the area model, and computing the product using the traditional algorithm. Students also compare and contrast various methods for dividing two- and three-digit numbers, such as ratio tables, mental arithmetic, and common long division algorithms. In the context of city blocks and miles, students multiply and divide whole numbers by fractions. They solve fraction division problems in other contexts and develop a strategy to divide mixed numbers by a fraction. The focus of Section C is to

- **calculate the product or quotient of two- and three-digit numbers;**
- **investigate a variety of strategies to solve multiplication and division problems including the standard algorithms;**
- **solve fraction division problems in a meaningful way; and**
- **develop number sense by operating on numbers in a convenient way.**

Learning Lines

Number Models: The Ratio Table and Area Model

In this section, students use the ratio table as a flexible tool to multiply and divide whole numbers. Starting with the ratio given in the problem— 1 crate to 24 bottles—students find equivalent ratios that build to a total number of crates (multiplication) or to a total number of bottles (division).

In addition, students use the ratio table to examine the intermediate steps in the standard algorithms for multiplication and division and describe what is actually calculated in each step. Because the quantities in the ratio table correspond to items in the original problem, the ratio table can help make sense of the series of steps in the algorithm. The area model method is similar to the traditional algorithm because it breaks up the multiplication of two-digit numbers into four separate problems. The four partial products are then added to find the final product. The area model was introduced in the unit *Facts and Factors.* For example, $61 \times 24 = (60 + 1) \times (20 + 4)$.

Operations with Fractions

To solve "bare" fraction division problems, for example, $3\ 1/4 \div 1/8$, students can either think of a context that fits the bare problem or use an algorithm. The division algorithm for fractions is also useful for dividing two mixed numbers. Note that in the unit *Fraction Times*, students found common denominators when they compared fractions. They used segmented bars or ratio tables for these comparisons.

Multiplication and Division

Multiplication and division are very closely related. For every multiplication problem, there are division problems related to it.

If $24 \times 49 = 1{,}176$, then $1{,}176 \div 24 = 49$, and $1{,}176 \div 49 = 24$.

This relationship is made explicit in Section D.

Revisiting Numbers

Section C Investigating Algorithms

Learning Outcomes

Students are able to find a product or quotient of two- and three-digit numbers. They understand how multiplication and division algorithms work, and they are able to solve fraction division problems.

Revisiting Numbers

Section D Operations

Section Focus

The focus of Section D is to investigate the concept that division by zero is undefined; develop an understanding of the relationship between multiplication and division; develop an understanding for multiplying and dividing integers; and understand and use the communicative, distributive, and associative properties.

Learning Lines

Operations: Multiplication and Division

Multiplication and division are very closely related. For every multiplication problem, there are two division problems related to it. For example, if $3 \times 7 = 21$, then $21 \div 7 = 3$ and $21 \div 3 = 7$.

Multiplication and Division of Integers

In the unit *Operations*, rules for addition, subtraction, and multiplication were made explicit. In Section D, students start to divide integers. To divide integers, the relationship with multiplication and the rules for multiplying integers can be used. For example: $-10 \div -2 = 5$ because $5 \times -2 = -10$.

Other explanations are $-10 \div -2 = 5$ because -2 fits 5 times into -10, or the "algebraic principle of permanence" can be used:

$10 \div 2 = 5$ $\quad$ $-10 \div -2 = 5$

$10 \div -2 = -5$ $\quad$ $-10 \div 2 = -5$

The four rules for division with integers are explored:

positive ÷ positive = positive
positive ÷ negative = negative
negative ÷ positive = negative
negative ÷ negative = positive

Properties

Students will have experiences with several properties without knowing the official names. For example, when you multiply, you can change the order of the factors and still get the same answer. For example, 8×2 is the same as 2×8. As a formal rule, this is written as $a \times b = b \times a$, and known as the *commutative property*.

Revisiting Numbers

Section D Operations

In Section D, students investigate whether the commutative, the distributive, and the associative property are valid for the different operations. In the Summary of Section D, these properties are formally written using variables:

- **Commutative property**
  - Of addition: $a + b = b + a$
  - Of multiplication: $a \times b = b \times a$
- **Distributive property**
  - Of multiplication over addition: $a \times (b + c) = a \times b + a \times c$
  - Of multiplication over subtraction: $a \times (b - c) = a \times b - a \times c$
- **Associative property**
  - Of addition: $a + (b + c) = (a + b) + c$
  - Of multiplication: $a \times (b \times c) = (a \times b) \times c$

Learning Outcomes

Students understand that division by zero is undefined. They have formalized rules for dividing integers, and they have developed an understanding of the commutative, the distributive, and the associative properties.

Revisiting Numbers

Section E Reflections on Numbers

Section Focus

The instructional focus of Section E is to
• explore integers, rational numbers, and irrational numbers and
• investigate the structure of the real number system.

Learning Lines

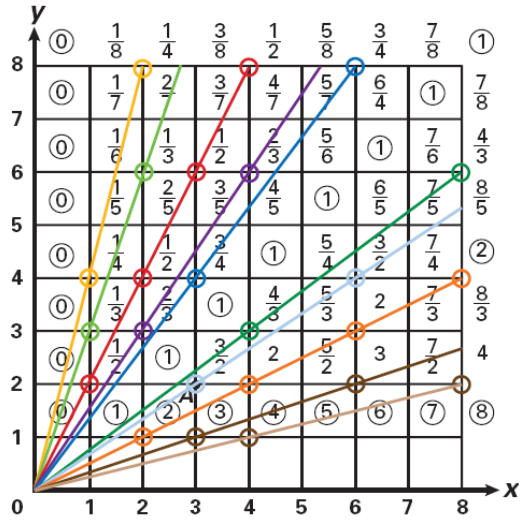
Number Systems and Number Sense

This section introduces students to the rules for operations with whole numbers (0, 1, 2, 3, 4, and so on) to explore the structure of the real number system (rational and irrational numbers). Students discover that adding and multiplying two whole numbers will always result in a whole number, that subtracting two whole numbers will always result in an integer (positive numbers, negative numbers, and zero), and that dividing two whole numbers will always result in a rational number (any number that can be written as the division of an integer by a non-zero integer) except when dividing by zero.

At the end of the section, students investigate taking the square and square root of whole numbers (natural numbers and zero) to explore the irrational numbers (a non-ending, non-repeating decimal—that is, a number that is not a rational number).

Ratio and Slope

The coordinate grid in which students obtain values for each grid point by dividing the first coordinate by the second is shown here. In the resulting number pattern, the same number appears at regular intervals in the grid. For example, if you start at any value 2 on the grid and you move right two and up one, you always arrive at another value of 2. The reason this occurs is because all the points with the quotient of 2 lie on a line with slope 1/2, with the line passing through the origin.



Revisiting Numbers

Section E Reflections on Numbers

Students will be introduced informally to the concept of slope in the unit *Looking at an Angle*. (*Steepness* is defined as the ratio between the height and the distance.) *Slope* is formally introduced in the unit *Graphing Equations,* where it is defined as the ratio of two numbers: the vertical change over the horizontal change.

Learning Outcomes

Students have developed an understanding of the structure of the real number system, including rational and irrational numbers.