# Overview for Families

*Mathematics in Context* unit: ***Reallotment***

Mathematical strand: **Geometry and Measurement**

The following pages will help you to understand the mathematics that your child is currently studying as well as the type of problems (s)he will solve in this unit.

Each page is divided into three parts:

- *Section Focus*
  Identifies the mathematical content of each section.

- *Learning Lines*
  Describes the mathematical flow of each section.

- *Learning Outcomes*
  Outlines what students should know and be able to do at the end of each section.

*"From the very beginning of his education, the child should experience the joy of discovery."*

Alfred North Whitehead

# *Reallotment*

## Section A The Size of Shapes

### Section Focus

**The instructional focus of Section A is to**

**• develop an understanding of the concept of area;**
**• compare, estimate, and compute areas of shapes using a variety of strategies and measuring units; and**
**• develop an understanding of the concept of reallotment.**

### Learning Lines

**Compare Area**
Students use estimation and informally use the concepts of ratio and proportion to compare areas of different-sized shapes. For example, students place one shape on the top of another and look at the overlapping sections. They also use unconventional units of measure, such as dot patterns, to compare the areas of shapes. (They estimate or count the numbers of dots in two shapes.) The strategies students use to compare the areas of shapes in this section not only are important in developing their understanding of area and their ability to determine area but also lay a foundation that will help them better understand how formal area formulas are derived in Section B.

**Area**
The concept of area—the number of measuring units needed to cover a shape—is implicitly introduced. The mathematical term *area* is not used until after students have experienced filling the interior of a two-dimensional shape.

**Tessellations**
Tessellations are patterns that fill a plane using congruent copies of a figure that do not overlap. For example, a tessellation can be made by covering a plane using regular triangles. A tessellation pattern can be created by beginning with a basic shape, cutting portions out of one part, and pasting them onto another part. These patterns reinforce the concept of reallotment, a powerful tool for estimating and computing area.

**Measuring Units**
First, students use a square tile with a given price as a measuring unit to determine the sizes and prices of tiles. The use of the square as a measuring unit is introduced as a mathematical convention and related to the relative cost of different-sized tiles. Then a rectangular grid is used to compare and estimate the areas of shapes. The number of measuring units needed to cover the shape is the area of a shape.

# *Reallotment*

## Section A The Size of Shapes

**Strategies**

The strategies that are used to find the number of square units that cover a shape generally deal with reallotting (or reshaping) a shape. A shape can be seen as the sum of other shapes or as a portion of another shape. A shape can also be rearranged to form a different shape by cutting and pasting. The strategies that students develop in this section are

- counting the number of whole squares in a shape and estimating the number of squares that the remaining pieces will make;
- subdividing the shape into parts that are easy to work with;
- reshaping the figure by cutting and pasting so that the area of the new shape can be found easily;
- enclosing the shape in a rectangle and subtracting the areas outside the shape; and
- using relationships between shapes.

For example, the area enclosed by a right triangle is exactly one-half of the area enclosed by a rectangle that has sides of the same lengths. This concept can also be used to find the area of other triangles.

## Learning Outcomes

Students have further developed their understanding of area, developed strategies to compare and estimate areas, and explored strategies for measuring the area of various shapes using squares as a unit for area.

# *Reallotment*

## Section B Area Patterns

### Section Focus

**The instructional focus of Section B is to**

**• develop students' understanding of the concept of area further;**

**• compare, estimate, and compute areas of shapes using a variety of strategies, such as using squares as measuring units; and**

**• develop students' formal vocabulary related to shapes and area, using words like *base*, *height*, *area*, and *square unit* and names such as *quadrilateral* and *parallelogram*.**

### Learning Lines

**Strategies**

The strategies for estimating and calculating area that students developed in Section A are made more explicit in this section:

• counting the number of whole squares in a shape and combining the remaining pieces into whole squares;

• reshaping the figure by cutting and pasting so that the area of the new shape can be found easily;

• enclosing the shape in a rectangle and subtracting the areas outside the shape;

• using relationships between shapes; and

• using formulas.

**Formulas: Area**

The use of base and height measurements leads to formulas for the areas of rectangles, parallelograms, and triangles. Counting strategies for finding area, such as the number of squares in one row times the number of rows, are an important base to develop students' understanding of the formula: $A_{rectangle} = base \times height$.

To derive a formula for the area of a parallelogram from the area of a rectangle, several strategies can be used, such as the compensating strategy or shifting. The compensating strategy means to cut and paste triangular sections of the parallelogram to reshape the figure into a rectangle. A diagonal of a parallelogram divides the parallelogram into two congruent triangles. Therefore, every triangle can be considered as half of a parallelogram:

$A_{triangle} = 1/2\ base \times height$.

### Learning Outcomes

Students are able to use informal and pre-formal strategies that involve transformations to estimate and compute the area of irregular shapes as well as the area of rectangles and other polygons. Students have further developed formal vocabulary and methods for calculating area. They start to use formulas for finding the area enclosed by rectangles, triangles, and parallelograms.

# *Reallotment*

## Section C Measuring Area

### Section Focus

**The instructional focus of Section C is to**

**• develop students' understanding of which units and tools are appropriate to estimate and measure length and area;**

**• develop students' understanding of and ability to use relations between units of measure within the metric system and within the customary system; and**

**• compare, estimate, and compute areas using metric and customary measuring units.**

### Learning Lines

**Measuring Units**

In this section, students use metric and customary units of measure. The difference between a linear unit of measure and a square unit of measure is addressed. The metric system is based on refinements in tenths. One millimeter is a refinement of one centimeter; it is one-tenth of a centimeter. Students studied refinements of units of measure within the metric system (i.e., meter, decimeter, and centimeter) in the unit *Models You Can Count On*. Studying metric units helps students develop an understanding of decimal numbers.

**Length**

Every student should have an idea about the relative sizes of 1 centimeter, 1 meter, and so on, in order to estimate lengths and to convert between metric units. For example, if students have points of reference for 1 meter and 1 centimeter, they can estimate that there are 100 centimeters in 1 meter. Your thumbnail is about 1 centimeter wide, which is smaller than one inch.

**Area**

To have an idea about the relative sizes of one square centimeter, one square meter, and so on, the dimensions of the shape are used. The relationships between square units of different sizes can be found by calculating how many smaller units fit into each larger unit. For example, one square meter has 100 rows of 100 square centimeters; therefore, a total of $100 \times 100$ square centimeters fit inside a square meter. In the same way, the relationships between customary square units of measure can be found: In one square yard, three rows of three square feet will fit; so one square yard is nine square feet. Students do not convert between the metric and customary systems.

**Area Model**

Finding the areas of rectangular floors with sides that are partly fractional, pre-formally addresses the multiplication of mixed numbers and fractions. This method is later used in other contexts for multiplying fractions, for example, in the context of probability, and is referred to as the area model. Using the area of a rectangle as a model to multiply mixed numbers is made explicit in the unit *Facts and Factors*. For example, to calculate $2\ 1/2 \times 3\ 1/2$, a drawing of a 2 1/2 by 3 1/2 rectangle is used as a model.

## *Reallotment*

### Section C Measuring Area

### Learning Outcomes

Students are able to use measurement units and tools appropriately for lengths and areas. Students know and can use relations between units of measure within the metric system and within the customary system.

# *Reallotment*

## Section D Perimeter and Area

### Section Focus

**The instructional focus of Section D is to**

**• develop students' understanding of the concept of perimeter;**

**• develop students' understanding of which units and tools are appropriate to estimate and measure perimeter and area; and**

**• develop students' formal vocabulary related to shapes, using words like *perimeter, diameter, radius,* and *circumference* and names like *equilateral triangle* and *regular hexagon*.**

### Learning Lines

**Area and Perimeter**

Just as activities like covering floors with carpet help to develop students' understanding of the concept of area, measuring trails around lakes or fences around gardens helps to develop students' understanding of the concept of perimeter. In this section, students explore relationships between perimeter and area. They discover that

• two shapes with the same perimeter can have different areas;

• two shapes with the same area can have different perimeters; and

• when a shape is enlarged, the area will enlarge more than the dimensions, or perimeter.

This relationship can be written formally as the following: If the factor of enlargement is $k$, then the factor of enlargement of the area is $k^2$. Factors of enlargement and reduction are made explicit in the unit *Ratios and Rates*.

### Learning Outcomes

Students are able to estimate and calculate the perimeter of a shape and its enlargement. They understand the effect of enlarging a shape. Students have used both informal as well as formal ways to find perimeters. Students can use formulas to compute the circumference and area of a circle.

# *Reallotment*

## Section E Surface Area and Volume

### Section Focus

**The instructional focus of Section E is to**

**• develop students' understanding of the concept of volume and surface area;**

**• develop students' understanding of which units and tools are appropriate to estimate and measure volume; and**

**• develop students' formal vocabulary related to shapes using terms like *surface area*, *volume*, *base*, and *cubic units*.**

### Learning Lines

**Surface Area and Volume**

To develop students' understanding of the concept of volume, they fill packages with cubes. The surface area is how much covering is needed to wrap all the sides of a package. Nets of packages are used to find the surface area.

In this section, students explore relationships between surface area and volume. They discover that shapes with the same volume can have different surface areas. To support the development of students' understanding of the concept of volume, finding the volume of a solid in cubic centimeters is related to the question, *How many centimeter cubes fit in the shape?* To find the volume of a shape, students will progress from

• informal, counting the number of centimeter cubes in one layer and then counting the number of layers, to

• pre-formal, the number of centimeter cubes in the length times the number of centimeter cubes in the width times the number of layers, to

• formal, $volume = length \times width \times height$.

Volume is expressed using both metric and customary units.

**Strategies**

In previous sections, students developed strategies for finding area, for example:

• dividing shapes into smaller parts whose area is more easily found;

• enclosing shapes in rectangles and subtracting the area of the "extra" parts; and

• using formulas.

Similar strategies can be used for finding volume:

• dividing solids into smaller parts whose volume is more easily found;

• enclosing solids in rectangular blocks and subtracting the volume of the "extra" parts; and

• using formulas.

### Learning Outcomes

Students are able to find the volume of shapes using informal as well as formal strategies. For example, they can find volumes by counting "unit" blocks or by reshaping irregular shapes. Students have developed an understanding of when and how to use the formula $Volume = area\ of\ slice \times height$. They understand the relationship between volume and surface area.