Overview for Families

Mathematics in Context unit: ***Ratios and Rates***

Mathematical strand: **Number**

The following pages will help you to understand the mathematics that your child is currently studying as well as the type of problems (s)he will solve in this unit.

Each page is divided into three parts:

- *Section Focus*
  Identifies the mathematical content of each section.

- *Learning Lines*
  Describes the mathematical flow of each section.

- *Learning Outcomes*
  Outlines what students should know and be able to do at the end of each section.

"From the very beginning of his education, the child should experience the joy of discovery."

Alfred North Whitehead

Ratios and Rates

Section A Single Number Ratios

Section Focus

Students explore ratios that express averages. They calculate the average number of people per car and the number of miles that a car can travel on a gallon of gas. They use ratio tables and calculators to write ratios as single numbers such as average speed (mi/h) or gas mileage (mpg). While using the ratio table, students also multiply and divide decimal numbers.

Learning Lines

Ratios, Averages, and Rates

In this section, students use ratios to express relationships between quantities and they explore and calculate single number ratios. In the context of carpooling, students use the ratio of people to cars, 12 people to 10 cars or 12:10, to find the average number of people per car. This average can be considered as a ratio in which something is compared to one (or a rate). This can be written as a single (decimal) number: "the average number of people per car is 1.2." Students use this average to determine other ratios of people to cars that are equivalent. The average as mean was introduced in the units *Picturing Numbers* and *Dealing with Data*.

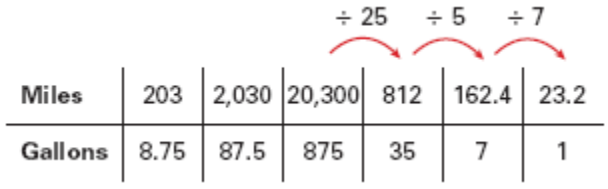
Students make calculations involving other single number ratios such as gas mileage (in mpg) and average speed (in mi/h). Students use the ratio table and a calculator to "scale the ratios to 1," and to compare ratios.

Models

The ratio table is a model used throughout *Mathematics in Context* to organize calculations with ratios, fractions, decimals, percents, and whole numbers. In this section, students use the ratio table to calculate averages and determine other equivalent ratios.

Miles	50	10	20	
Gallons	2.5	0.5	1	

Students are introduced to an intuitive informal strategy for working with decimal numbers in a ratio table. They repeatedly multiply both quantities by ten until the ratio includes only whole numbers. Students can then scale back the ratio to miles per gallon by dividing.



The ratio table provides a way to model the problem and the steps in the solution process.

Ratios and Rates

Section A Single Number Ratios

Learning Outcomes

Students use ratios to express relationships between quantities, write a ratio as a single number to express an average, and use ratio tables to solve problems involving ratios. Students begin to use ratio tables to compare ratios and understand how rates or ratios can be used as an average.

Ratios and Rates

Section B Comparisons

Section Focus

Students make relative and absolute comparisons of the number of people and telephones or television sets in different countries. Students reflect on the appropriateness of using each type of comparison. They use ratios as averages and write ratios (to one) as single numbers.

Learning Lines

Absolute and Relative Comparisons

This section introduces problems dealing with absolute and relative comparisons. Students compare, for example, the number of telephones in different countries and relate the number of telephones to the population for each country.

Students work with large numbers, and often the words *million* and *billion* are used to write these numbers. Students compare these numbers where they must be careful to interpret the magnitude indicated by million or billion.

Sometimes ratios can be calculated in two "directions": the number of telephones per person or the number of people per telephone. Depending on the context and the problem, one of these ratios can be more appropriate or meaningful. Relative comparisons are made using ratios, often written as single numbers that are ratios per one (averages), but also ratios "per 100" or "per 1,000" are used for relative comparisons. Ratio tables can be very useful for making calculations.

Models

Ratio tables are useful models for making relative comparisons. Students use the ratio tables to solve problems involving comparisons. They work with large numbers, where *million* and *billion* are used as a "measuring unit." Students continue to work with decimal numbers in ratio tables.

Learning Outcomes

Students use ratio tables to compare ratios, know the difference between absolute and relative comparisons, and use the notion of rate or ratio as an average to make relative comparisons. Students also compare large numbers and use million and billion as a kind of measure, for example, when they compare 150 million to 1.2 billion.

Ratios and Rates

Section C Different Kinds of Ratios

Section Focus

Students investigate the relation between part-part ratios and part-whole ratios. They express part-whole ratios as fractions, percents, and decimals.

Learning Lines

Ratios, Fractions, Percents, and Decimals

In the context of speeding cars, students explore the difference between part-part and part-whole ratios. If, for example, two cars are speeding for every eight that are not, this is a part-part ratio 2:8. The whole is not mentioned. It is also possible to say, "Two out of every ten cars are speeding." This is a part-whole ratio, 2:10, or "two out of 10." If students want to use ratios to make comparisons, the ratios must be of the same kind, either part-part or part-whole. Students learn to turn part-part into part-whole ratios. Only part-whole ratios can be expressed as fractions, percents, and decimals.

Relationships among fractions, percents, and decimals are revisited in this section. Students have seen this before in the units *More or Less* and *Fraction Times*. The relationship with ratios is formally introduced here. Students find equivalent ratios, fractions, decimals, and percents. Students use these relationships to write ratios as percents. They can, for example, use benchmark fractions.

Models

Students use ratio tables to convert ratios to percents by rewriting a ratio as a number compared to 100 or "per 100."

Learning Outcomes

Students can

- relate ratios to fractions, decimals, and percents;
- use relationships among ratios, fractions, percents, and decimals to solve problems involving ratio;
- understand problems with part-part ratios versus part-whole ratios;
- write (part-whole) ratios as percents; and
- use ratio tables and a calculator to solve problems involving ratios.

Ratios and Rates

Section D Scale and Ratio

Section Focus

Students explore scale drawings and scale models in several contexts. They use statements like "1 cm on a map represents 1,000 cm in reality," scale lines, and scale ratios to express scales. Students use double number lines and ratio tables to make calculations involving scale.

Learning Lines

Ratios and Scale

A scale is a constant ratio between actual measurements and measurements in the enlarged or reduced representation. If a room is drawn to a certain scale, all furniture must be drawn on the same scale as well.

A scale ratio shows the relationship between the dimensions in the drawing and the actual dimensions of the object. A scale ratio of 1:100 on a floor plan can mean

- 1 centimeter represents 100 centimeters or
- 1 meter represents 100 meters or
- 1 millimeter represents 100 millimeters or
- 1 inch represents 100 inches.

Students find the scale of scale drawings by calculating the ratio between the actual measurements and the measurements on the scale drawing.

Models

The double number line is used to show the relationship between actual dimensions and dimensions of a scale drawing. Scale lines on maps can be transformed into double number lines as well. This can help students correctly interpret a scale on a map. It can also help finding actual distances.

The ratio table is used to organize the work and make calculations involving scale. A difference between the ratio table and the double number line is that on the double number line, the numbers are always placed in numerical order whereas in a ratio table this is not necessary.

Learning Outcomes

Students understand scale as a constant ratio between actual dimensions and dimensions in a reduced representation and find the scale ratio using an actual length and the reduced size. Students also use centimeters or millimeters to measure distances and review relationships between metric units, for example, between meters and centimeters and between kilometers and meters. They start to make connections between scale ratios and scale lines on maps and use a scale line on a map to calculate actual distances.

Ratios and Rates

Section E Scale Factor

Section Focus

Students explore scale factors for enlargements and reductions. They use scale factors to find dimensions of the original object or of the reduction or enlargement. Students also determine the scale factor when they are given the length of a real object and an enlarged or reduced drawing of the object. They relate scale ratio and scale factor. They use arrow language and ratio tables to work with scale factor.

Learning Lines

Ratios and Scale Factors

This section introduces scale factors to solve problems involving enlargements and reductions. Scale factors are introduced in the context of enlarged and reduced drawings of animals. A scale factor indicates how all measurements of an object have been enlarged or reduced. Scale factors are constant ratios—all dimensions of an object are enlarged or reduced by the same factor. A scale factor is always a multiplier. Students learn that if a scale factor is greater than one, it represents an enlargement; if a scale factor is between zero and one, it represents a reduction.

Arrow language and ratio tables are used to work with scale factors. Students were introduced to arrow language in the unit *Expressions and Formulas*. Students learn how to use arrow language and ratio tables to determine the scale factor if the dimensions of the original and the reduction or enlargement are known.

Students use the scale factor to determine measurements of the original object from the measurements of the reproduction and vice versa. In solving this type of problem, students use connections between operations and fractions; for example, if the scale factor of a reduction is 0.25, students can use the fact that length $\times$ 0.25 is the same as a length $\div 4$ to make the calculation easier. The relation between multiplication and division is informally addressed in this section while working with arrow language

Models

Arrow language is used to represent scale factors. Students can use arrow language to make calculations. From the unit *Expression and Formulas*, students know how to reverse arrow strings to make calculations. They know that division "undoes" multiplication. Students use ratio tables to find the scale factor. They connect the scale ratio and the scale factor through the use of ratio tables.

Learning Outcomes

Students understand scale factor. They identify scale factors in situations of enlargement and reduction and know that scale factors larger than one belong to enlargements, while scale factors between zero and one represent reductions. Students can use a scale factor to find actual sizes in situations of reductions and enlargements and find the scale factor using an actual length and an enlarged (or reduced) size. They also use arrow language and ratio tables to solve problems involving scale factors.