Overview for Families

Mathematics in Context unit: ***Patterns and Figures***

Mathematical strand: **Algebra**

The following pages will help you to understand the mathematics that your child is currently studying as well as the type of problems (s)he will solve in this unit.

Each page is divided into three parts:

- *Section Focus*
  Identifies the mathematical content of each section.
- *Learning Lines*
  Describes the mathematical flow of each section.
- *Learning Outcomes*
  Outlines what students should know and be able to do at the end of each section.

"From the very beginning of his education, the child should experience the joy of discovery."

Alfred North Whitehead

Patterns and Figures

Section A Patterns

Section Focus

In this section, students use recursive and direct formulas to represent number sequences and dot patterns. Recursive formulas are used to generate the next term in the number sequence using the current term. Direct formulas, on the other hand, generate any term in the sequence given the pattern number. The term *expression* will be used in later sections to refer to direct formulas without the equal signs.

Note: Students have used recursive and direct formulas in previous Algebra units.

Learning Lines

Patterns and Regularities

Students discuss color patterns on number strips and extend the patterns to describe large numbers. For example, on a number strip with three alternating colors, students use the multiples of three to determine the color of a large number on the number strip.

Next, students investigate the use of dot patterns and formulas to represent the color patterns on the number strip. After writing formulas for the even and odd number sequence, students investigate recursive formulas for these patterns and review the terminology for recursive and direct formulas. Next, they investigate W and V dot patterns by analyzing the relationship between the pattern number and the number of dots. They then write recursive and direct formulas for W and V dot patterns.

Models

Students investigate the use of dot patterns to represent number sequences.

Learning Outcomes

Students use and create dot patterns, number strips, or charts to visualize number sequences. They also use and create recursive formulas to describe number sequences

Patterns and Figures

Section B Sequences

Section Focus

This section introduces the concept of *arithmetic sequence*. Students use repeated addition and subtraction to generate other terms in a given sequence.

Learning Lines

In an arithmetic sequence, the next numbers in the sequence are generated by a constant increase or decrease. Students investigate the step-by-step increase/decrease on number strips, and they identify if a sequence is an arithmetic one or not. In case of arithmetic sequences, students will then demonstrate their ability to extend an arithmetic sequence and write recursive and direct formulas to represent them.

Students add and subtract number strips and expressions for arithmetic sequences to generate new arithmetic sequences. Students start to combine the number strips of the odd and even numbers to form a new number strip. Then they investigate adding and subtracting the number sequences and the expressions that represent the sequences. Students will find that the constant increase for the new sequence is the sum of the increases for the two original sequences. Students then combine the algebraic expressions that represent the sequences to form the expression for the new sequence.

In this section, Euler's formula is used as an example of combining sequences: Students make number strips and write direct formulas to represent the number of vertices, faces, and edges in a sequence of pyramids and combine the strips and expressions to form Euler's formula

Models
Number strips help to visualize sequences and operations between sequences.

Learning Outcomes

Students identify and extend an arithmetic sequence, and write recursive and direct formulas to represent an arithmetic sequence. Students will also add and subtract number sequences and the corresponding expressions.

Patterns and Figures

Section C Square Numbers

Section Focus

In this section, students investigate the sequence of square numbers by investigating number sequences, geometric patterns, and an area model. Students generate the square numbers from a tile pattern and then represent the sequence in a number strip. They use square dot patterns to explain the increases in the number strip and use the pattern of increases to extend the sequence of square numbers. Students then use an area model to represent the square of a number and write related expressions. (Square numbers were also addressed in the unit *Facts and Factors*.)

Learning Lines

Number Sense

Students determine the dimensions of the largest square patio that can be made using 200 square tiles that measure 30 centimeters by 30 centimeters. Later they will determine how many squares they can make using the 200 tiles.

Patterns and Regularities

Students add number strips for the odd and square number sequences. They discover that the resulting sequence is the square number sequence without the zero term (shifted up by one) and can be represented by the expression $(n + 1)^2$. Next, students use an area model to investigate squaring the expression for the $(n + 1)$ sequence and add number strips to form the $(n + 2)^2$ sequence. Students then represent the $(n + 2)^2$ sequence with an area model. They then demonstrate their ability to describe the square number sequence and use an area model to square algebraic expressions. Students generalize a concrete number sequence to an expression that describes the sequence.

Models

Students use visual models to extend their understanding of equivalent expressions and formulas. For example:

- Students use square dot patterns to explain the increases in the number strip.
- Students use an area model to square numbers; for example, a number like 32 is broken into 30 and 2 and then used to label two sides of a square. The area model generates an expression with four terms—in this case: $(30 \times 30) + (30 \times 2) + (30 \times 2) + (2 \times 2) = 900 + 60 + 60 + 4 = 1024$.
- Students use an area model to investigate squaring the expression $(n + 1)^2$.

Learning Outcomes

Students understand the sequence of square numbers. They justify equivalent expressions. They use visual models to extend their understanding of equivalent expressions and formulas. They create and use expressions and direct formulas to describe number sequences.

Patterns and Figures

Section D Triangles and Triangular Numbers

Section Focus

Students apply the square number sequence to solve problems involving a tessellated triangle pattern. Students name basic polygons and investigate a sequence of tessellated equilateral triangles. They discover that the total number of triangles in the pattern can be represented by the square number sequence. Students then rearrange the white and red triangles in the pattern to show that the number of tiles in the tessellation is equal to the square of the number tiles in the base.

Learning Lines

Patterns and Regularities

Students investigate the connections between triangle tessellations, dot patterns, and triangular numbers. Then using the dot pattern for the rectangular numbers, students justify two different formulas for the rectangular numbers and demonstrate that each rectangular number is double a triangular number. Students write a formula for the triangular numbers and apply this formula to solve problems involving a stack of cans. Students extend a table that lists the number of ping-pong matches played at a competition with various numbers of players. After listing the number sequence and drawing a diagram to represent the context, students write a direct formula that they can use to calculate the number of ping-pong matches played in the competition with any number of players.

Learning Outcomes

Students use triangular and rectangular numbers as examples of describing sequences. They also use formulas and expressions to describe patterns and sequences in realistic situations.