Overview for Families

Mathematics in Context unit: ***Packages and Polygons***

Mathematical strand: **Geometry and Measurement**

The following pages will help you to understand the mathematics that your child is currently studying as well as the type of problems (s)he will solve in this unit.

Each page is divided into three parts:

- *Section Focus*
  Identifies the mathematical content of each section.

- *Learning Lines*
  Describes the mathematical flow of each section.

- *Learning Outcomes*
  Outlines what students should know and be able to do at the end of each section.

"From the very beginning of his education, the child should experience the joy of discovery."

Alfred North Whitehead

Packages and Polygons

Section A Packages

Section Focus

The instructional focus of Section A is to

- **identify and sort three-dimensional shapes by category;**
- **compare the properties of three-dimensional shapes;**
- **construct three-dimensional shapes from two-dimensional nets;**
- **identify two-dimensional shapes from three-dimensional representations; and**
- **develop spatial visualization skills.**

Learning Lines

Paper Models and Nets

Nets are a way of visualizing solids and help to identify properties of solids. A net is a two-dimensional figure that can be folded into a three-dimensional solid. Nets are used to explain the concept of a face of a shape. Students learn that the properties of the two-dimensional shape (such as the length and the width of each face) are extended to three dimensions (such as height) when the net is folded.

Students are informally introduced to the concept of height, which is further explored in Section E. Using nets as models enables students to create definitions of, to conjecture about, and to gain insight into geometric relationships.

Spatial Sense

The visualization aspect of this section is important. A sense of shape depends, to a large extent, on mental images. For instance, any two-dimensional picture of a three-dimensional solid requires the viewer to infer information about the parts of the solid that are hidden. The hands-on activities in this section will help to develop students' spatial sense. When students have to interpret two-dimensional drawings of three-dimensional situations, some students will be able to do this easily; students who struggle can use paper models to support their visualization. Students are encouraged to solve problems at a level that makes sense. For example, when a student cannot visualize the process of folding a net, they can turn to the concrete form and cut the net out to make the polyhedron.

Learning Outcomes

Students recognize and identify geometric shapes and structure in real objects and in representations and know their names. They have experience in constructing geometric models and are able to make connections between different views of geometric solids. They develop spatial visualization skills and start to solve three-dimensional problems using two-dimensional representations (nets) or reasoning.

Packages and Polygons

Section B   Bar Models

Section Focus

In this section, students construct bar models of polyhedra. They identify and count the vertices and edges on their bar models and test their models for stability. Students learn by investigating that triangles are very stable figures. They draw hidden edges and vertices in pictures of prisms and pyramids; they draw all face diagonals and space diagonals in a cube, and they count them for other solids. Throughout this section, students answer questions and solve problems regarding the structure of pyramids and prisms. The instructional focus of Section B is to

- **construct three-dimensional bar models of various three-dimensional shapes;**
- **identify the edges, vertices, and faces of three-dimensional shapes;**
- **investigate the stability of triangular shapes;**
- **learn the concepts of face diagonal and space diagonal; and**
- **develop spatial visualization skills.**

Learning Lines

Bar Models

Bar models nicely show the concept of edges (the drinking straws or toothpicks) and the concept of vertices (where the bars meet, the gumdrops). The bar models help students understand the property of rigidity. For example, triangles are rigid, but rectangles are not. Using this principle, a cube can be made more stable by adding extra bars on faces or within the solid. This leads to the introduction to the concepts of face diagonal and space diagonal.

Students identify which solids have face diagonals and space diagonals. When they have to find the number of face diagonals and the number of space diagonals, they may develop a systematic way to count them. Students' experience with bar models will be extended in Section D, where they study the Platonic solids.

Spatial Sense

Also in this section, the visualization skills play a role, for example, when students construct bar models and reflect on how many materials are needed or when they draw hidden edges and vertices in three-dimensional pictures. Again, the hands-on activities in this section will help to develop students' spatial sense. When students have to interpret two-dimensional drawings of three-dimensional situations, some students will be able to do this easily; students who struggle can use bar models (or the paper models they made in Section A) to help them visualize the situation.

Learning Outcomes

Students recognize and identify properties of geometric shapes. They have experience in constructing geometric models and are able to make connections between different views of geometric solids. They can identify edges, vertices, and faces, and they understand the concepts of face diagonal and space diagonal. They develop spatial visualization skills and solve three-dimensional problems using two-dimensional representations or reasoning.

Packages and Polygons

Section C Polygons

Section Focus

Students explore the characteristics of polygons by investigating the number of ways a shape fits onto itself. They learn what a polygon is and identify several common polygons. They compare regular and irregular polygons, considering what makes some polygons regular. Students investigate the exterior and interior angles and develop strategies to find the measure of the interior angles of any regular polygon. The instructional focus of Section C is to

- **identify the properties of regular polygons and**
- **develop strategies to find the measure of the interior angles of a polygon.**

Learning Lines

Two-Dimensional Shapes

The word *polygon* comes from the Greek word *polygonos*, which means "many angled." Polygons are two-dimensional closed shapes with three or more angles.

By exploring turns and thinking about how a shape fits onto itself, students discover that regular polygons have equal angles and equal sides and so can "fit" onto themselves in more ways than non-regular polygons. If you have an n-sided regular polygon, it can fit onto itself $2n$ ways (allowing flips).

In Section D, the concept of polygons and regular polygons is used to define polyhedra and regular polyhedra, or Platonic solids.

Measurement

Students may remember from the unit *Figuring All the Angles* that the angles of a triangle total 180º. In that unit, students learned to measure angles using a compass card. They also used this skill to measure angles in regular polygons. Students learned about the relationships between turns and inside or resulting angles. If you make a turn, the turn and the resulting angle add up to 180º. Turn can perhaps be best understood by thinking about walking around the edge of a polygon. When you arrive back at the point of origin, you have turned 360º.

For regular polygons, the size of the turn, or exterior angle, can be found by dividing 360º by the number of turns. (For example, in a regular pentagon, $360^\circ \div 5 = 72^\circ$.) Since the sum of the size of a turn and the resulting (or interior) angle is 180º, each interior angle can be found by subtracting the size of the turn from 180º ($180^\circ - 72^\circ = 108^\circ$).

In addition to the strategy of measuring using a compass card or making turns, a third strategy is possible for a hexagon: Join three hexagons to form a honeycomb pattern. Three angles meet to form 360º. The angles are equal, so $360^\circ \div 3 = 120^\circ$ per angle.

Learning Outcomes

Students recognize and identify the properties of regular polygons and identify polygons as two-dimensional shapes and have some experiences in drawing polygons. Students use the relationships between angles and turns to solve problems. They can choose a strategy to find measurements of the angles of a regular polygon.

Packages and Polygons

Section D   Polyhedra

Section Focus

The instructional focus of Section D is to

• construct models and investigate properties of regular polyhedra and semi-regular polyhedra;

• investigate the relationship between the number of faces, vertices, and edges for regular and semi-regular polyhedra;

• apply Euler's formula to various polyhedra; and

• develop spatial visualization skills.

Learning Lines

Three-dimensional Shapes

Polyhedra are three-dimensional shapes whose faces are all polygons. The word *polyhedron* comes from the Greek words for "many bases." In this section, students use their experiences from the previous sections to explore the properties of the five regular polyhedra, also known as the Platonic solids. Students are not expected to give a formal definition, but they should be able to see that all the faces of a regular polyhedron are congruent regular polygons and that the same number of edges intersect at each vertex.

Learning Outcomes

Students identify regular and semi-regular polyhedra. They understand and use Euler's formula. Students develop spatial visualization skills and solve three-dimensional problems using two-dimensional representations (nets and drawings), three dimensional representations (paper and bar models), or reasoning. Students develop efficient counting strategies involving geometric solids, which can be generalized.

Packages and Polygons

Section E Volume

Section Focus

The instructional focus of Section E is to
- **further develop the concept of volume;**
- **investigate and understand the relationship between liters and cubic centimeters;**
- **use strategies that involve transformations of three-dimensional shapes to estimate and compute volume;**
- **calculate the volume of three-dimensional shapes using formulas;**
- **measure the height of a three-dimensional shape; and**
- **start to find the volume of pyramids and cones.**

Learning Lines

Volume
In the units *Reallotment* and *Made to Measure*, students investigated the concept of volume. Essential to this understanding is the following: When students are asked to find the volume of a rectangular block, do they know that they have to find the answer to *How many cubic units fit in the shape?* Students will likely begin this section with different levels of understanding volume.

Students revisit the formula *Volume = area of slice × height*. They investigate when this formula can be correctly applied. For cylinders, the formula can be written as:

$$Volume_{cylinder} = \pi \times radius \times radius \times height$$

Often this formula is presented as:

$$Volume_{cylinder} = \pi \times r^2 \times h$$

Learning Outcomes

Students further develop their concept of volume. They understand and use the relationship between liters and cubic centimeters. They use strategies that involve transformations of three-dimensional shapes to estimate and compute volume. Students measure the height of a three-dimensional shape and understand and use the formula:

$$Volume = area\ of\ slice \times height$$

They calculate the volume of a cylinder, using the formula:

$Volume = area\ of\ slice \times height$, or $Volume = \pi \times radius \times radius \times height$

They start to calculate the volume of a pyramid and a cone, using the formulas:

$Volume_{pyramid} = 1/3$ *of the volume of a prism with the same base and height*

$Volume_{cone} = 1/3$ *of the volume of a cylinder with the same base and height*