

Overview for Families

Mathematics in Context unit: **Operations**

Mathematical strand: **Algebra**

The following pages will help you to understand the mathematics that your child is currently studying as well as the type of problems (s)he will solve in this unit.

Each page is divided into three parts:

- *Section Focus*
Identifies the mathematical content of each section.
- *Learning Lines*
Describes the mathematical flow of each section.
- *Learning Outcomes*
Outlines what students should know and be able to do at the end of each section.

“From the very beginning of his education, the child should experience the joy of discovery.”

Alfred North Whitehead

Operations

Section A Positive and Negative

Section Focus

We expect students to be aware of the existence of negative numbers without computing with positive and negative numbers. Positive and negative numbers are revisited within the context of time difference from one location to another and the elevation places on the earth below and above sea level. The first context involves, for example, Americans watching the live broadcast of a tennis match in Australia. Students use a cylindrical model of the earth with positive and negative numbers to represent the time zone as an introduction to the use of the number line. The number line is reviewed within the context “above and below sea level” where students investigate the difference between highest and lowest points on the earth. Informal addition and subtraction of integers starts with counting on the number line.

Learning Lines

Number Sense

Students review the concept of integers using a variety of contexts. Within the contexts used in this section, they start ordering numbers on the number line. Informal addition and subtraction is introduced by counting on the number line within a context. Zero is neither positive nor negative. Students model a situation involving integers by using a concrete model for time zones.

Learning Outcomes

Students are able to model situations using positive and negative numbers.

Operations

Section B Walking Along the Number Line

Section Focus

As the title suggests, the focus of Section B is the number line. The number line is connected to the time zone model from Section A that indicates deviations from Greenwich Mean Time. Number lines are used in a variety of forms—horizontal, vertical, and curved—and even a “real” one, as represented by a rope or tape on the classroom floor. Pre-formal addition and subtraction of integers are introduced with the help of a robot game. Ronnie the Robot walks along the number line while following students’ instructions about adding and subtracting. For example, with either instruction, ADD -8 or SUBTRACT 8, Ronnie will take 8 steps in a negative direction.

Some statements that are made about a number line used in mathematics:

- A number line can be extended in both directions infinitely.
- If the number line is horizontal, the positive numbers are to the right of zero and the negative numbers are to the left of zero.
- If the number line is vertical, the positive numbers are above zero and the negative numbers are below zero.
- If you move along the line in the positive direction, the numbers you pass become greater and greater.
- If you move along the line in the negative direction, the numbers become lesser and lesser.
- Decimals and fractions, both positive and negative, can also be indicated on a number line.

Learning Lines

Number Sense

Students learn to locate and order integers on several types of number lines. On both a context-based and a context-free number line, students show that although five is less than ten, negative five is more than negative ten. Students also use formal notation ($<$, $>$, and $=$) for comparing and ordering integers. Initially, words rather than symbols are used for adding and subtracting integers to avoid confusion about the dual use of the negative sign as an operator and as an indicator of a negative number.

Models

These models of a real number line and the physical experience of being a robot walking forward and backward while carrying out instructions help students get a better understanding of the operations with integers. The formalized number line is used as a model for ordering, adding, and subtracting integers.

Learning Outcomes

Students are able to order integers on a number line and use the formal notations for *less than* ($<$) and *greater than* ($>$). They identify the difference between two numbers, for instance 45 and -10 by looking at the distance on the number line from one number to the next. No formal notation for distance is used. Students add and subtract integers in a pre-formal way. Some students may still prefer to use the robot model.

Operations

Section C Calculating with Positive and Negative Numbers

Section Focus

In this section, students practice addition and subtraction of integers in a more formal way. Through this section, the references to operations change from language-based (informal) to symbolic (formal). The language-based notation from the previous section is reviewed. Then circles are used:

$$\begin{array}{r} (-32) + (68) = (36) \\ (32) + (-68) = (-36) \end{array}$$

Parentheses make it possible to discern negative numbers in a calculation instead of the circles used earlier: $14 - (-3) = 17$ and $14 + (-3) = 11$.

Learning Lines

Number Sense

Students show their knowledge of the concept of operations with integers on a reflective level by answering questions like: *Is this statement always true? Explain why or why not.*

Repeated addition is written as a multiplication, for example, $(-5) \times (-5) \times (-5) = 3 \times (-5) = -15$. To become flexible in carrying out operations with integers, students practice while filling in numbers in addition and subtraction “trees.”

Models

The number line is used to strengthen the concept of adding and subtracting integers. The calculation $-32 + 68$ is carried out by splitting 68 into 32 and 36. First add 32 to -32 and arrive at 0 on the number line; then add the remaining 36.

Learning Outcomes

Students practice adding and subtracting integers using parentheses instead of circles to discern the operation from negative numbers. They develop formal strategies to multiply integers.

Operations

Section D Adding and Multiplying

Section Focus

Addition, subtraction, and multiplication of integers are formalized in this section. Division of integers is informally introduced in this unit and addressed formally in the units *Revisiting Numbers* and *Algebra Rules!* (In Algebra units, division of negative numbers is addressed with the computation of *slope*.) In this unit, students anticipate the division of negative numbers as they complete multiplication trees.

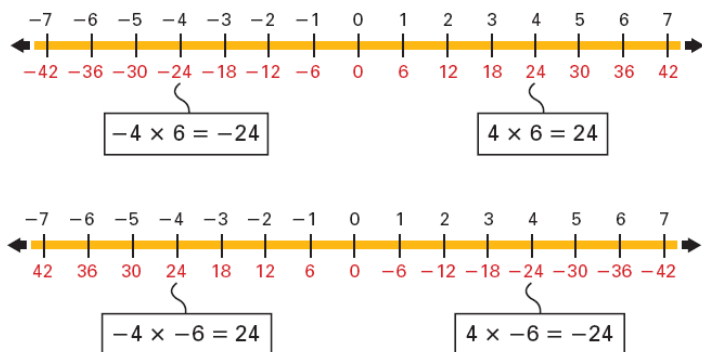
Students practice working with integers within the context of finding a mean in a data set. This method of computing the mean for a data set of numbers is a smart way to do so without having to calculate the sum of all numbers. First estimate a mean from the data set. Then find the difference from this mean for each entry and calculate the mean of the differences. (Students could also cancel out opposite differences from the estimated mean.) To find the mean, add (or subtract) the mean of the differences from the estimated mean. The mathematics behind this principle is that when you add (or subtract) the same number to or from all numbers in a data set, the mean changes by the same amount.

Learning Lines

Number Sense

In order to explain the rules for multiplication with negative numbers, two methods are used, double number lines and the “algebraic principle of permanence.”

Double number line:



Algebraic principle of permanence:

| | | | |
|---------------------|--------------------------------|---------------------|---------------------------|
| $3 \times 6 = 18$ |) For each step, subtract 6 | $3 \times -6 = -18$ |) For each step, add 6 |
| $2 \times 6 = 12$ | | $2 \times -6 = -12$ | |
| $1 \times 6 = 6$ | | $1 \times -6 = -6$ | |
| $0 \times 6 = 0$ | | $0 \times -6 = 0$ | |
| $-1 \times 6 = -6$ | | $-1 \times -6 = 6$ | |
| $-2 \times 6 = -12$ | | $-2 \times -6 = 12$ | |
| $-3 \times 6 = -18$ | | $-3 \times -6 = 18$ | |

In this section, the four rules for multiplication are made explicit. Parentheses to discern negative numbers are not used all the time anymore.

Operations

Section D Adding and Multiplying

Learning Outcomes

Students apply and use the formal rules for multiplication of integers. They combine the operations adding, subtracting, and multiplying in a variety of problems.

Operations

Section E Operations and Coordinates

Section Focus

Informal introduction of the coordinate system — only positive numbers — has taken place in the unit *Expressions and Formulas*. Students review the positive coordinate system by using the map of Provo, Utah, from *Figuring all the Angles*, to give greater visual coherence between units. In Section E, the coordinate system is expanded with negative numbers. Students plot and interpret points on this coordinate system. They practice using the rules for addition, subtraction, and multiplication of integers within the context of transformations of geometric shapes on the coordinate system.

Learning Lines

Number Sense

Students apply operations with positive and negative numbers. By using transformations of geometric shapes on the coordinate system, they find that *negative* \times *negative* = *positive* within a new context.

Models

Operations with integers are applied within the model of the coordinate system. A coordinate system is used to locate points on a grid by indicating two coordinates that tell where the point is located relative to the origin.

The position of a geometric shape, drawn on a grid, can be changed by adding a number to or subtracting a number from each coordinate (or one of them). Multiplying or dividing the coordinates by some number is also possible. Students need to apply the rules for the operations with integers to do this, reinforcing their knowledge of the concept of integers. Visualization of the effects of performing operations with integers is a powerful way to understand the concept.

Learning Outcomes

Students are able to plot and read points from a coordinate system with four quadrants. They apply the operations with positive and negative numbers by investigating transformations of geometric shapes on the coordinate system.