Overview for Families

Mathematics in Context unit: ***Models You Can Count On***

Mathematical strand: **Number**

The following pages will help you to understand the mathematics that your child is currently studying as well as the type of problems (s)he will solve in this unit.

Each page is divided into three parts:

- *Section Focus*
  Identifies the mathematical content of each section.

- *Learning Lines*
  Describes the mathematical flow of each section.

- *Learning Outcomes*
  Outlines what students should know and be able to do at the end of each section.

"From the very beginning of his education, the child should experience the joy of discovery."

Alfred North Whitehead

Models You Can Count On

Section A The Ratio Table

Section Focus

The focus of this section is the introduction of the ratio table and the development of students' ability to generate new numbers in the table. The operations that students can use are made explicit: adding, times 10, doubling, subtracting, multiplying, and halving.

Learning Lines

Ratios

This section builds on students' informal knowledge of ratios and part-whole relationships. The context of recipes supports the development of the concept of ratios.

The section starts with recipes in order to help students develop their informal understanding of ratios.

Models

The ratio table that is introduced in this section is more a tool than a model; however, it is a powerful tool for solving ratio problems, and it helps students to develop their understanding of ratios further.

Number Sense and Computational Skills

Students will use and further develop their number sense and computational skills when they generate equivalent ratios and carry out operations like doubling, halving, times ten, and multiply and divide by a number. All the operations that students use in a ratio table, they can also use for the models that are developed in the other sections: the fraction and percent bars in Section B and the double number line in Section D.

Decimals

In the beginning of this section, students work with whole numbers and generate larger numbers; later they generate smaller numbers when they have to find the price per unit. The context of money offers the opportunity to reinforce computations with decimal numbers. Students' understanding of decimal place value will be further developed in Sections C and D.

Learning Outcomes

Students will have developed strategies to generate new numbers in a ratio table (equivalent ratios). They will be able to use a ratio table as an organizational tool and will have developed a conceptual understanding of ratio.

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Section B The Bar Model

Section Focus

This section focuses on fractions and the relationship between fractions and percents. Students' understanding of these concepts is supported by the development of the fraction bar and the percent bar model.

Learning Lines

Fractions

In this section, fractions arise as a result of a division, in a fair share situation. For example, if you equally divide something among four persons, each will get one out of four, or 1/4. Students can visualize this concept using a measuring strip.

Most fractions that occur are benchmark fractions, which are "easy" fractions, such as 1/2, 1/4, 1/8, and 1/5. Students will use these fractions to estimate a quantity to a given whole. In this way, they informally use a fraction as an operator. Students will make more computations with fractions in Sections C and D.

Percents

The percents used in the beginning are benchmark percents, such as 50%, 25%, and 10%, and they are related to the benchmark fractions. In the context of tipping in a restaurant, students start to use a percent as an operator when they find a percentage of a certain amount.

Ratios

In the context of water tanks and gauges, students start to make relative and absolute comparisons informally.

The development of the fraction bar and the percent bar takes place in specific contexts. Students use these bar models in contexts such as dividing plots, readings gauges of a water tank (the fraction bar), and solving problems related to a download bar on a computer (the percent bar). The fraction bar as well as the percent bar gives the students visual support to make computations and estimations. The strategies students developed using a ratio table in Section A can be applied when using these bar models.

Learning Outcomes

Students will be able to represent and make sense of calculations involving fractions and percents using a bar model (percent bar). Students will understand that a fraction is the result of a division and a description of a part-whole relationship. They are able to connect benchmark percents (e.g., 1%, 10%, 25%, 33%, and 50%) to fractions. They combine benchmark percents to find other percents (for example, using 10% and 5% to find 15%).

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Section C The Number Line

Section Focus

The focus of this section is to introduce the number line as a model to locate places using fractions in the context of distances on a map. Students are introduced to the empty number line model to make simple computations with decimals. This section also focuses on decimal place value and on comparing and ordering decimals.

Learning Lines

Models

The number line model helps students develop a conceptual understanding of fractions as numbers. The empty number line is a tool that is based on the number line: The numbers are placed in the correct order, but not necessarily to scale.

Fractions

Students use their experiences from Section A to order, compare, and reason about the location of (benchmark) fractions on a number line. In Section D, students will solve context problems where they operate with fractions informally.

Decimals

In this section, students' understanding of place value, their ability to compare and order numbers with one decimal is revisited and further developed. A fraction bar gives visual support for the decimal notation of tenths. A method for adding numbers uses an empty number line. By moving from one number to another in the fewest jumps and by jumping only 0.1 space, 1 space, or 10 spaces at the time, students are learning to add and subtract decimals mentally. This method is known as the *compensation method*. For example, to jump from 1.6 to 2.5 on the number line, jump 1 to 2.6, and then jump 0.1 back ($2.5 - 1.6 = 0.9$).

Students' understanding of decimal numbers with two decimals is supported by the context of money. Changing dollar amounts into cents is a powerful strategy to help understand decimal place value. Students' implicit knowledge of decimals will be expanded in Section D.

Learning Outcomes

Students will have further developed a conceptual understanding of fractions and decimals. They order and compare fractions and decimals on a single number line and use fractions as numbers and as measures.

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Section D The Double Number Line

Section Focus

This section focuses on the double number line as a visual model to use for ratio problems. In this section, students make sense of calculations involving fractions and decimals, and they use informal strategies to multiply and divide fractions and decimals using a double number line.

Learning Lines

Models

The double number line model allows students to make accurate calculations and estimates for many types of ratio problems. The operations that students use on a double number line are similar to the operations they learned in Section A when they used a ratio table. Instead of a double number line, a ratio table could also be used. However, a double number line gives visual support: The numbers are ordered. Note that a double number line can start at zero, but a ratio table cannot.

A double number line, in general, is a visual model used to represent the equivalencies between two units. A special type of a double number line is the *double scale line,* as used on a map.

Fractions

In the context of city blocks and miles, students' understanding of fractions as part-whole relationships is further developed. Students informally operate with fractions (additions, multiplications, divisions) supported by various problem contexts. For example, to solve the problem "How many city blocks are in 1 1/4 miles?" students use the double number line to find 1 1/4 ÷1/8.

Decimals

In Section D, the ordering of decimal numbers from Section C is reviewed. Students place weights (one decimal) and prices (two decimals) on a double number line, make estimations, and make accurate calculations. The context of money provides students the option of converting from dollars to cents to eliminate the decimal point. After solving the problem, if necessary, students can convert cents back into dollars.

Ratios

Students informally work with ratios and rates such as the following: One city block is 1/8 mile, and in 10 minutes, you can comfortably walk 1/3 mile.

Learning Outcomes

Students use scale lines and maps to determine distance and use double number lines to relate travel time and distance. Students represent and make sense of calculations involving fractions and decimals using a double number line. They informally multiply and divide fractions using a double number line.

Models You Can Count On

Section E Choose Your Model

Section Focus

Students solve problems and decide which model they want to use. This section also focuses on metric units and their relationships.

Learning Lines

Models

All of the number models from Sections A–D are reviewed in this section: ratio table, fraction bar, percent bar, number line, and double number line.

Measurement: Decimals

In Section C, students worked with miles and tenths of a mile. One-tenth of a mile has no special name, but one-tenth of a meter does: *deci*meter. In the context of the long jump, students review their use of the number line as a model and build their understanding of numbers given in tenths and hundredths.

The metric measurement units (i.e., meter, decimeter, and centimeter) are used to develop students' understanding of decimal place value further and to introduce conversions between meters, decimeters, and centimeters. The context of measurement units (combined with a number line) may also offer the opportunity to reinforce students' computation skills and their ability to solve problems involving decimal numbers. Later, students can refer to this context when they have to solve context-free problems. For example, to find a number between 2.95 and 3, students can think of 2.95 meters and 3.00 meters, which are 295 centimeters and 300 centimeters.

Learning Outcomes

Students choose their own model to solve problems involving ratios, proportions, fractions, decimals, and percents.