

Overview for Families

Mathematics in Context unit: ***It's All the Same***

Mathematical strand: **Geometry and Measurement**

The following pages will help you to understand the mathematics that your child is currently studying as well as the type of problems (s)he will solve in this unit.

Each page is divided into three parts:

- *Section Focus*
Identifies the mathematical content of each section.
- *Learning Lines*
Describes the mathematical flow of each section.
- *Learning Outcomes*
Outlines what students should know and be able to do at the end of each section.

“From the very beginning of his education, the child should experience the joy of discovery.”

Alfred North Whitehead

It's All the Same

Section A Tessellations

Section Focus

Students investigate similar shapes by tessellating congruent shapes to form large, similar shapes. In addition, they find algebraic rules for the number of triangles in each row of a triangle tessellation in relation to the total number of triangles that completely fill the large triangle. The properties of parallel lines, used to break down triangle tessellations, are reviewed. Parallel lines and congruent triangles were introduced in the unit *Triangles and Beyond*. Tessellations help students to understand similar triangles in several ways. They further develop the concept of ratio. This mathematical model also helps to find lengths of sides of similar triangles. And finally, the model helps students to develop a deeper understanding of the relationship between parallel lines and corresponding angles, which will be used later in the unit to prove similarity formally.

Learning Lines

Characteristics and Properties of Shapes

Students review the concept of congruent triangles without formal proof. They also review the concept of parallel lines and investigate how families of parallel lines can be used to tessellate a triangle.

Geometric Relationships

Students investigate the relationship between the number of rows in a triangle tessellation and the number of triangles along each side of a triangle and find a formula to represent this relationship.

Transformations

The formal term is not used in this section. However, tessellations are an example of informal transformations such as flips, turns, and slides.

Using Visualization, Spatial Reasoning, and Geometric Modeling to Solve Problems

Students use tessellations to find a relationship between an isosceles triangle and a rhombus, each tessellated by the same triangles.

Learning Outcomes

Students can say in their own words what parallel lines and congruent triangles are. They have informal knowledge of similar triangles without using the mathematical term yet. They have informally compared areas of similar shapes formed by different tessellations.

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Section B Enlargement and Reduction

Section Focus

Similar triangles are investigated in a pre-formal way with the help of enlargements and reductions. Students explored this concept in the unit *Ratios and Rates*. The ratio table is used here to help students organize calculations when finding lengths of corresponding sides of similar triangles. Students find that not all problems about similar triangles can be solved using tessellations. In this section, the multiplication factor is used to describe the relationship between shapes that are either an enlargement or a reduction of each other. A connection is made with percents to represent a multiplication factor. Students learn what a multiplication factor is; for example, 300% means the factor of enlargement is 3. In this section, no formal proof of similarity is expected of students yet, but the formal term is used.

Learning Lines

Characteristics and Properties of Shapes

Students learn another method to describe similar triangles, by way of a multiplication factor. They analyze overlapping triangles and identify corresponding sides. Then they use the relationship between those sides to find the length of an unknown side.

Geometric Relationships

Students further investigate the relationship between the ratios of side lengths and areas of similar triangles.

Transformations

Students review transformations as used in tessellations and identify equal angles.

Using Visualization, Spatial Reasoning, and Geometric Modeling to Solve Problems

Students choose the tessellation strategy for similar triangles or use the multiplication factor to find the ratio of corresponding sides in similar triangles to solve problems involving the dimensions of a ramp of a bridge. They use similar triangles to find the length of a book shelf in a triangle-shaped bedroom under a roof top and make assumptions for missing information.

Learning Outcomes

Students choose between two strategies to solve problems involving similar triangles. They know how to use the ratio of corresponding sides, the multiplication factor. They understand that a shape is enlarged if the multiplication factor is greater than 1 and reduced if the multiplication factor is smaller than 1 but greater than zero. They can express multiplication factors as the ratio of corresponding sides, using fractions, whole numbers, and percents.

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Section C Similarity

Section Focus

In this section, the concept of similar shapes is formalized. Formal mathematical notations and formal mathematical language are used. Students create similar and non-similar triangles with specified dimensions and observe that similar triangles have equal angle measures. In the previous sections, students were not expected to prove triangles were similar; in this section, they learn how to do that. There are two ways to prove that two triangles are similar:

- Two triangles are similar if all corresponding sides have the same ratio. In order to draw this conclusion, you have to know the ratios of all corresponding sides.
- Two triangles are similar if all the corresponding angles are equal. To draw this conclusion, it is sufficient to know the measurements of two corresponding angles of each triangle.

Learning Lines

Characteristics and Properties of Shapes

The notion of acute, obtuse, and right angles is reviewed. Students review how parallel lines intersected by another straight line show angles with equal measures. This concept was addressed earlier in the unit *Triangles and Beyond*. Students learn that all circles are similar shapes and all squares are similar shapes, but not all rectangles are.

Learning Outcomes

Students know formal rules to prove whether or not triangles are similar. They can choose their own strategy to solve problems involving similar triangles. They can identify equal angles produced by parallel lines and an intersecting line.

It's All the Same

Section D Similar Problems

Section Focus

In this section, what students learned in previous sections is connected, reviewed, and expanded. Patterns of right triangles within squares are explored, and the areas of similar triangles in these designs are compared. Students apply their knowledge of the properties of similar triangles while solving a variety of realistic problems. They find the width of a porch attached to a house, they investigate special effects for early motion pictures, and they decide whether or not a stepladder can be used on the floor space available for it.

Learning Lines

Characteristics and Properties of Shapes

Students use the Pythagorean theorem to calculate side lengths of a right triangle. They review the classification of triangles and decide which type of triangle—isosceles or equilateral—is used for a tessellation. They use the properties of squares and rectangles to create geometric patterns.

Geometric Relationships

Students use similarity of triangles to solve a variety of problems.

Transformations

Students review transformations as used in tessellations and identify equal angles.

Using Visualization, Spatial Reasoning, and Geometric Modeling to Solve Problems

By using similar triangles, students solve various realistic problems.

Learning Outcomes

Students know formal rules to prove whether or not triangles are similar. They can choose their own strategy to solve problems involving similar triangles. They are able to reason mathematically about situations involving similar triangles.

It's All the Same

Section E Coordinate Geometry

Section Focus

In this section, the contents of the previous sections as well as the contents of previous units are now formalized. The concepts of parallel and perpendicular lines are used to introduce students to formal mathematical reasoning and proof. The concept of slope was addressed earlier in the Algebra units and was formalized in *Graphing Equations*. Students practice finding the slopes of lines that are drawn in a coordinate system. They use coordinate geometry to represent and examine the properties of geometric shapes. They prove that a rhombus has equal side lengths and that the diagonals of a rhombus are perpendicular to each other. The Pythagorean theorem is used to calculate the distance between two points in a coordinate system. The distance formula, based on the theorem as well, is not used here.

Learning Lines

Characteristics and Properties of Shapes

Students recall that parallel lines have equal slopes, and they now learn that if lines are perpendicular, their slopes have the opposite sign and are reciprocal numbers. Students learn (and prove) that a rhombus has equal side lengths and the diagonals of a rhombus are perpendicular. They use their own strategy to show a triangle has a right angle. They learn the properties of a kite (i.e., two pairs of equal sides, diagonals are perpendicular).

Geometric Relationships

Students use the Pythagorean theorem to calculate the distance between two points in a coordinate system.

Using Visualization, Spatial Reasoning, and Geometric Modeling to Solve Problems

Students use similar triangles to solve problems within the context of coordinate geometry.

Learning Outcomes

Students have used the Pythagorean theorem and its inverse. They can use the theorem to calculate the distance between two points in a coordinate system. They are introduced to more formal mathematical reasoning and proof within the context of coordinate geometry. For example, they can prove whether or not a quadrilateral is a rectangle, a rhombus, or a parallelogram.