Overview for Families

Mathematics in Context unit: ***Great Predications***

Mathematical strand: **Data Analysis and Probability**

The following pages will help you to understand the mathematics that your child is currently studying as well as the type of problems (s)he will solve in this unit.

Each page is divided into three parts:

- *Section Focus*
  Identifies the mathematical content of each section.

- *Learning Lines*
  Describes the mathematical flow of each section.

- *Learning Outcomes*
  Outlines what students should know and be able to do at the end of each section.

"From the very beginning of his education, the child should experience the joy of discovery."

Alfred North Whitehead

Great Predictions

Section A Drawing Conclusions from Samples

Section Focus

Students investigate chance and expected outcomes in several real-world situations. They discover that drawing conclusions from samples always involves uncertainty. Students consider how samples should be taken to be able to draw reliable conclusions. They study situations in which samples give unusual results, and they reflect on whether there is a cause for these unexpected outcomes or whether they may be due to chance.

Learning Lines

Sample and Population

In this section, the terms *sample* and *population* are important concepts. These were introduced and used in the unit *Dealing with Data* and further elaborated and used in the units *Insights into Data* and *Second Chance*. In this section, the terms are revisited and formalized. Students use results from samples to make estimates about the population using ratios and percents. They investigate several real-life situations, such as television ratings, surveys for favorite music, and attendance in the House of Representatives.

Students consider how reliable the estimates for the population are, whether the sampling process was biased, and if unexpected results may be due to chance or other factors. Students understand that sampling needs to be random, representative, and not selective (e.g., only asking friends). A sample also needs to be of sufficient size. Students also realize that, in real-life situations, other information than the information collected by sampling may be needed to make a decision about a situation.

Sample and Simulation

Students select random samples from a population of 400 colored and white squares. The squares are a model of the real-life situation, and students simulate taking samples. In the simulation, students can take as many random samples as they like and as large a sample as they like. In doing so, students discover that sampling always involves uncertainty and there is variability in the results. By taking several samples or by increasing the sample size, the chance of getting unexpected and unusual results gets smaller and the results will better reflect the population.

Learning Outcomes

Students have a better understanding of the relationship between a sample and a population. They can draw conclusions from samples, and they know that uncertainty or chance is involved when doing this. They are also aware of methods to reduce this uncertainty. Students know that samples must be large enough and randomly selected to give representative results for the population.

Great Predictions

Section B Maybe There Is a Connection

Section Focus

In this section, students evaluate results from surveys, samples, or experiments to determine whether or not there is a connection between apparently unrelated events. For example: *Is there a connection between where people live and how they vote? Does a new brand of insect repellent work and does it work better in some regions than in others? Does an orangutan in a zoo prefer blocks of a particular shape and color?* Students use tree diagrams, two-way tables, and chance trees to decide whether two events are independent or dependent.

Learning Lines

Dependent and Independent Events

An opinion poll about building a bridge across a river introduces the concepts of dependent and independent events. Students organize the data into a two-way table and a tree diagram to decide whether there is a connection between where a person lives and how a person votes.

Dependent and independent events were informally studied in the unit *Second Chance*; these concepts are revisited and formalized in this section. Some of the contexts, like wearing glasses from the unit *Second Chance*, are revisited in this section, and the issue of dependency is formalized. Students are made aware of the fact that although using models such as tree diagrams, two-way tables, and chance trees can support decisions about whether two events are dependent, these models cannot show why a connection exists or what type of connection exists between the variables studied.

Models

Students use tree diagrams, chance trees, and tables to organize information and make inferences about whether events seem to be dependent or not. These models were used in previous units in the Data Analysis and Probability strand. The chance tree that was introduced and used in the unit *Second Chance* is now expanded: in addition to chances (now written as percents), the absolute numbers are now added to the tree. This information makes it easier to calculate expected value, which is explored in the next section.

Learning Outcomes

Students understand the difference between independent and dependent events. They can organize information in two-way tables, tree diagrams, or chance trees to decide whether events are dependent or independent. Students understand that if events are dependent, this does not tell why a connection exists. This relationship may be due to chance or to other unknown factors.

Great Predictions

Section C Reasoning from Samples

Section Focus

In this section, students study the distribution of results from samples and compare this to the distribution in the population. They graph data in number-line plots and histograms. They compare data from several samples by comparing the plots and using statistical measures such as median and spread. They investigate what happens when samples get larger. They make chance statements based on data from samples and population.

Learning Lines

Sample and Population

In this section, the relation between samples and the population is further explored. Students simulate catching fish from a pond to determine the average length for different types of fish. Students learn that small samples can have much variability. Recording the results of growing samples, for example, by combining the results of all the samples taken by the whole class helps students realize that variability can be reduced as the sample size increases. A larger sample is also more likely to be representative of the population. The data from the samples (i.e., the type of fish and its length) are graphed in number line plots and histograms. Students were introduced to these types of graphs in the units *Picturing Numbers* and *Dealing with Data*.

Students notice that as the sample gets larger the distribution of the lengths of the two types of fish is more distinct. They use the sample data to verify claims made about the lengths of the two types of fish. Later they check this against the population data and conclude how well the sample data reflect the data of the population.

Graphs and Chances

In the unit *Second Chance,* students used data presented in graphs, like histograms, to estimate chances. They interpreted the relative frequency of an outcome as the experimental (empirical) chance for that outcome. This connection between data analysis and probability is further explored in this section.

Instead of calculating the relative frequencies from a histogram, students may also estimate chances by looking at the distribution of the outcomes to make more general statements about chance.

Students compare distributions of data for backpack weight for several samples of students from different grades. These data are graphed in number line plots; students use the median and the spread to describe the distributions.

Learning Outcomes

Students understand how to describe the distribution of data and make statements about experimental chance based on relative frequencies of certain outcomes. They improve their understanding of the relationship between a sample and a population by studying the effect of a growing sample.

Great Predictions

Section D Expectations

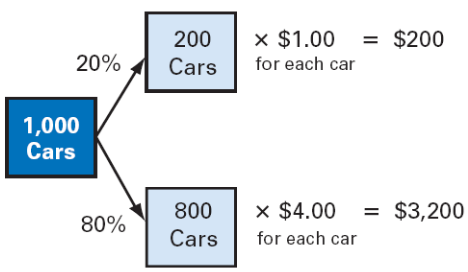
Section Focus

The concept of expected value is formalized in this section. Students calculate, for example, the expected revenue for several toll options, the expected number of readers that will become customers, and the expected score for a basketball player. Students use chance trees to model situations and compute probabilities. They relate chance and expected value.

Learning Lines

Expected Value

The concept of expected value, informally addressed in the unit *Second Chance*, is now formalized in this section. Chance trees, which were introduced in the unit *Second Chance*—as a special type of tree diagram—and were used in previous sections of this unit, are now used to explore and calculate expected values. To make the computation of expected value easier to understand, "absolute" numbers are added to the chance trees, as in the example shown.



The total expected amount of tolls is found using the chance tree. The average expected value can be found by dividing the total of tolls collected, which is $3,400, over the 1,000 cars. This results in an expected toll of $3.40 per car. The average expected value can also be calculated by multiplying an outcome by its chance and adding the results for the several outcomes. In the example of the toll road, this would be $0.2 \times \$1.00 + 0.8 \times \$4.00 =$ $3.40 per car. This method is not explicitly addressed in this section. Students explore what happens if they start with a different number of cars, and they discover that the expected value only depends on the chances of the different outcomes. Expected value and mean are related when students calculate the expected life span of mayflies.

Models

Students use the chance tree with absolute numbers to calculate expected values. This chance tree was introduced in Section B in this unit. Students also use data presented in tables to calculate expected value. To do so, they must first reorganize the information.

Learning Outcomes

Students understand the meaning of expected value and can calculate and use it to make decisions. They use chance trees with absolute numbers to calculate expected value. They can also calculate expected value from data in a table. Students informally connect expected value to the (mathematical) mean.

Great Predictions

Section E Combining Situations

Section Focus

In this section, students investigate situations in which two events occur and the chance of each event is known. They use diagrams, area models, and chance trees to investigate both dependent and independent events. Students calculate chances for combined events. The multiplication rule for chance is formalized. This section reviews topics addressed in the unit *Second Chance* and in previous sections of this unit.

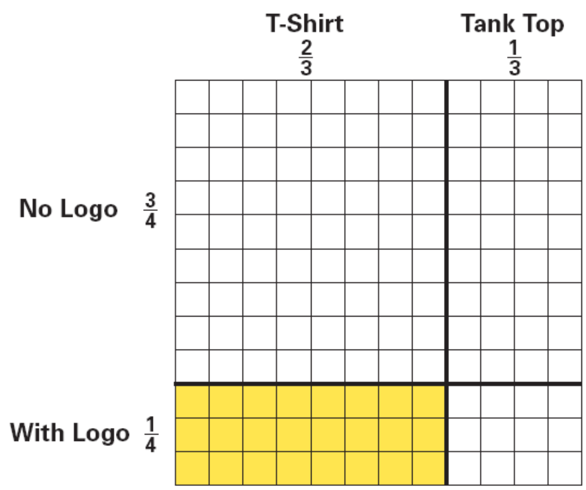
Learning Lines

Combined events

Students continue the study of chances in combined event situations that they began in the unit *Second Chance*. They compare experimental chances for the combined event of getting a free drink and/or a free hot dog to the theoretical chance. They explore more complex situations using chance trees and area models. The multiplication rule for chances is formalized. Students find out that this rule will not always work, such as in the case of dependent events.

Models and Strategies

Students can structure multi-event situations by using a chance tree or an area model. These models were introduced and used in the unit *Second Chance*.



In the illustration above, the area of the whole rectangle in the area model represents a chance of 1 as well as the total number of shirts that the problem is about (in this example 144). The area of each of the four parts into which the rectangle is divided represents the chance for the different combined outcomes. The chances of these outcomes can be found in two ways. For example, the chance of the combined event that a randomly chosen shirt is a T-shirt with a logo is 24 out of 144 (using the absolute numbers of small squares), which is 1/6. Or it can be said to be 1/4 × 2/3 (using the chances, the fractional parts), which is also 1/6.

Great Predictions

Section E Combining Situations

Using the chance tree, a similar method can be used: calculating with the absolute numbers in the boxes or using the chances along the branches. For both models, when using the chances instead of the absolute numbers, the multiplication rule for chances is applied. Students discover that this rule can be used only if the events are (or can be made) independent.

Learning Outcomes

Students can calculate chances in multi-event situations using chance trees or an area model The multiplication rule for chances (for independent events) is formalized in this unit.