# Overview for Families

*Mathematics in Context* unit: ***Graphing Equations***

Mathematical strand: **Algebra**

The following pages will help you to understand the mathematics that your child is currently studying as well as the type of problems (s)he will solve in this unit.

Each page is divided into three parts:

- *Section Focus*
  Identifies the mathematical content of each section.

- *Learning Lines*
  Describes the mathematical flow of each section.

- *Learning Outcomes*
  Outlines what students should know and be able to do at the end of each section.

*"From the very beginning of his education, the child should experience the joy of discovery."*

Alfred North Whitehead

# *Graphing Equations*

## Section A Where There's Smoke

### Section Focus

**Students use compass directions and degree measurements to plot lines to locate forest fires on a map. Later in the section, students also use coordinates to locate these fires on a computer screen that uses a four-quadrant coordinate grid. Students explore how the ($x$, $y$) positions change as the fire moves in different directions. They use equations to describe the movement of fires along lines. Students draw firebreaks that follow parts of horizontal and vertical lines, and use inequalities to describe regions enclosed by firebreaks.**

### Learning Lines

**Directions and Locations**

Describing directions is an important concept in geometry. In this unit, directions are connected to algebra, specifically to straight lines. Directions can be described in words, symbols (N, E, S, W), degrees, and with other measurements such as with coordinates relating to a coordinate grid system, or with slope.

**Compass Directions and Degrees**

Students are familiar with the eight wind or compass directions such as north (N) or southwest (SW) from earlier geometry units. This type of direction, starting from a point (the fire tower), is now used to locate forest fires. Students realize that two directions from different starting points (fire towers) are needed to locate a fire. A more refined way of giving directions is to divide the circle even more, into 360°. Students use degrees, beginning with 0° and measuring clockwise up to 360°, to locate fires. Based on given directions, students show the locations of fires on the map by intersecting the lines in the given directions.

**Coordinates**

The ($x$, $y$) notation in a coordinate grid system is another way of finding locations. It differs from the ways for using wind directions and degrees in that it does not give a direction. Students were introduced to the coordinate grid system in the unit *Operations*. Students locate fires and plot and find points with given coordinates in all four quadrants. Students use the formal vocabulary connected to the coordinate system.

**Directions and Coordinates**

Coordinates and directions are also combined: For a given location in ($x$, $y$) notation, students must be able to find the correct directions in degrees from different fire towers. (See, for example, problem 10 on page 4.) Directions involving the coordinate grid instead of degrees will be introduced in the next section, as students are introduced to the concept of slope.

*Graphing Equations*

## Section A  Where There's Smoke

### Lines and Regions

Locating fires from fixed positions (the fire towers) provides students with experiences that lead to the equations of vertical and horizontal lines, and later in the unit, to slope and equations of other straight lines. The moving of a fire along vertical and horizontal lines leads to equations such as $x = 10$ for a vertical line, which is the set of all points whose $x$-coordinate is the same, and $y = 8$ for a horizontal one (the $y$-coordinate is fixed). In other Algebra units, students have seen and used equations of lines, as arrow formulas, or written as equations with word-variables. In this unit, the more formal notation using $x$ and $y$ is used. Vertical and horizontal lines forming the boundaries of a rectangular region in which a fire spreads lead to the definition of a region by using inequalities; inequalities can be described in words or by using inequality signs. In this section, a region is described in three ways:

- in the context: the region north of the firebreak (line) at $y = 8$;
- in words: $y$ is greater than 8; and
- in symbols: $y > 8$.

Also two-sided inequalities such as $16 < x < 18$ are used.

## Learning Outcomes

Students are able to describe and graph directions using wind directions and angles. They understand how to find the intersection point of two lines graphically. Students can also give coordinates of a labeled point in a coordinate grid and reversely show the location of a point with given coordinates. They can describe and graph horizontal and vertical lines using their equations and can use inequalities to describe regions restricted by horizontal and vertical lines.

# *Graphing Equations*

## Section B Directions as Pairs of Numbers

### Section Focus

**In the previous section, students located fires using directions on a compass rose or using degrees. They also used *(x, y)* notation on a coordinate grid map to indicate the location of a fire. The fires were all located relative to the origin (0, 0). In this section, students describe directions in a grid map by using a horizontal and a vertical component. Students start by giving directions using pairs of numbers. They then learn to describe the direction as the slope of a line using a single number, the ratio of the vertical component to the horizontal component.**

### Learning Lines

**Directions as Pairs of Numbers**

In the previous section of this unit, directions were given using wind directions or angle measures in degrees. In the direction pair [+10, +5], the first number gives the horizontal component of the direction, and the second number gives the vertical component. In the context of the forest fire, this direction pair means "go 10 km east and 5 km north." Directions are not given relative to the origin (0, 0) but are relative to any starting point.

Different direction pairs describe the same direction, for example, [+10, +5] and [+20, +10]. Both positive and negative numbers as well as rational numbers are used in direction pairs. In order to avoid confusion between coordinate pairs that indicate a location relative to the origin (0, 0), the notation for direction pairs is different from the notation for coordinate pairs. Direction pairs use "straight" brackets [like these] and a sign for each number.

Since all direction pairs that indicate the same (or opposite) direction have the same ratio, this ratio can be used to describe the direction or slope of a line. Slope is introduced as the ratio of the vertical component of a direction pair divided by the horizontal component.

**Directions, Slope, and Equivalent Fractions**

The fact that different direction pairs indicate the same direction and thus also must generate the same ratio or slope can be used to address the notion of equivalent fractions. For example, using direction pairs, students should be able to show why $4/6 = -4/-6$ or why $-4/2 = -2$.

**Lines and Slope**

The slope of a line gives the "direction" of a line, which is a measure of how steep it is. It is given as the ratio of two numbers, the vertical change over the horizontal change. A line's slope is the same regardless of what two points on the line are used to compute it or what direction pair is used to find it. Students have to find the slope of a line from two given points or from a drawing. Students can try to find a direction pair for a line first and use this to calculate the slope. Students must draw a line if the slope and a point on the line are given. In Section C, the slope component for the equation of a line is discussed.

## *Graphing Equations*

### Section B Directions as Pairs of Numbers

### Learning Outcomes

Students will understand that different direction pairs can indicate the same direction and will produce the same slope. Students will understand the meaning of *slope* in different contexts. They will be able to find the slope using a direction pair, two points on a line, and/or a drawing. Students will also be able to draw a line if a direction pair or the slope and a point on the line are given.

# *Graphing Equations*

## Section C An Equation of a Line

### Section Focus

**Students use the direction of a line (described with a number pair) to explore how a line is drawn by taking bigger or smaller steps. This leads to a rule for finding the *y*-value of a point on a line after a certain number of horizontal steps. This rule leads to a formula for the equation of a line, with a starting point (later the *y*-intercept) and a slope. Students interpret the meanings of numbers in equations, and draw the lines described by equations. They describe the steepness of a line by measuring the angle the line makes with the *x*-axis. Students investigate when the tangent of that angle is the same as the slope of the line.**

### Learning Lines

**Directions and Steps**

In the previous section, students have investigated the use of direction pairs to describe directions. They have seen that different pairs may describe the same direction and that all points that lie in a given direction or its opposite starting from the same point form a line. Now students investigate how you can move along a line by taking steps in a certain direction. Taking horizontal steps of +1 leads to a rule to find the coordinates of points on the line, for example:

Starting point: (0, 5).
After 100 horizontal steps of + 1:
$x = 100$
$y = 5 + 100 \times 2 = 205$

These rules lead to a formula for a line relating the *x*-coordinates and the *y*-coordinates. For the example above, this formula is: $y = 5 + 2x$.

**Equation of a Line**

In this section, equations for lines are introduced. Students have seen equations of lines in other *Mathematics in Context* units. But these were mostly equations using word variables that relate to the context. Here the equations are more formally introduced and written in $y$ and $x$. A common way to write the equation of a line is in the form: $y = mx + b$, where the variable $m$ is the slope and the variable $b$ is the $y$-intercept. Starting from steps along a line, students investigate the role of the numbers in a rule for finding coordinates. They do the same for the numbers in the resulting equation by relating the equation to the rule and to the graph. In this way, they learn about the slope (see also Section B) and the $y$-intercept.

The $y$-intercept is where a line crosses the $y$-axis. This concept is formally introduced in this unit. Algebraically, the $y$-value of the equation equals the $y$-intercept when the $x$-value is 0. The slope indicates how steep a line is. Students draw lines for given equations and write equations for drawn lines.

# *Graphing Equations*

## Section C An Equation of a Line

**Slope and Tangent**

In this section, students investigate if the tangent of the angle that a line makes with the positive (or right) side of the $x$-axis is equal to the slope. This is the case only if both the horizontal and the vertical axes are scaled in the same way. The tangent of an angle is defined as the vertical distance divided by the horizontal distance. Students discover that the slope is not proportional to the angle.

## Learning Outcomes

In this section, students develop many skills: At the end, they can find the equation of a line from the graph of a line, graph a line given its equation, and investigate angles related to slope. They can use and find equations of lines in the coordinate plane in the form: $y = mx + b$, and they understand the meaning of slope and $y$-intercept.

# *Graphing Equations*

## Section D Solving Equations

### Section Focus

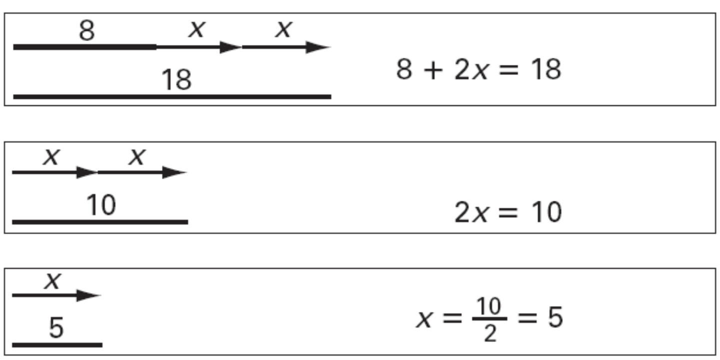
**Students investigate solving equations using diagrams, number lines, and symbols. They first do so informally in a context operation involving two imaginary frogs. The unknown that students have to find is the length of one single frog jump. The context helps set up a model involving diagrams and equations, which leads to a method for solving linear equations by canceling values on either side of the equation or otherwise stated: by performing the same operation on both sides of the equation.**

### Learning Lines

**Solving Equations—Informally Using Diagrams and a Number Line**

In the context of Alice and Fred, two frogs who are jumping away from a path, students consider what information they would need to determine Alice's and Fred's distances from the path after both have made a number of jumps of equal length from a given starting position. A diagram is used to clarify and structure the situation. If both Alice and Fred end at the same distance and take jumps of equal length, the length of such a jump can be found. This can be done informally by reasoning from the situation. Students solved systems of equations informally in earlier units such as the units *Comparing Quantities* and *Expressions and Formulas*.

The length of the jump, the unknown, can be represented by a variable, in this case $x$. This leads to an equation for the problem: $8 + 5x = 18 + 3x$. The $x$ can appear in both the diagram as well as the equation.




Students first use the diagram, still related to the context, to solve the problem of finding the length of each jump (the value of $x$). These diagrams actually visualize equations. In the context, it makes sense to cancel overlapping jumps and distances in the diagrams. Simultaneously with the changes in the diagrams during the solving, the equations change accordingly. This prepares students for the more formal way of solving equations by operating on the symbols.

Not only are equations with addition signs in them modeled with diagrams of the frog, subtraction can also be modeled if the frog jumps in the opposite direction. Another model is also used to represent and solve equations: jumps on the number line. This model allows for both positive and negative starting points (the numbers in the equations) as well as for positive and negative direction of jumps (the sign for the $x$-part in the equation).

# *Graphing Equations*

## Section D  Solving Equations

**Solving Equations—Formally**

By simultaneously changing the diagrams and the equations the diagram visualizes to solve a problem, students learn to understand and use a formal way of solving equations. In an equation, the same number can be added or subtracted from both sides without changing the answer (the $x$-value). Also, the same number can be multiplied or divided from both sides without changing the answer. Students learn to write down the operations they perform to keep track of the steps they take in solving the equations.

## Learning Outcomes

Students can solve equations of the form $a + bx = c + dx$, where $a$, $b$, $c$, and $d$ can be positive or negative. Students can choose an appropriate way to solve equations: use a diagram, a number line, or the formal method of operating on the symbols. Students know how to visualize an equation and reversely how to write an equation for a problem presented visually.

# *Graphing Equations*

## Section E Intersecting Lines

### Section Focus

**Students make the connection between the graphic and algebraic methods of solving equations (or finding the point of intersection of two lines). Students found the point of intersection of two lines in a graph in Sections A, B, and C in the context of forest fires. In Section D, students learned a formal algebraic method for solving linear equations by performing the same operation on both sides of the equation.**

### Learning Lines

**Point of Intersection and Solving Equations**

In Sections A and B, students found the point of intersection of two lines by drawing them or reading the coordinates from the graph. In Section C, students have to find a point of intersection from a graph, where the grid is too small to show this point. Students need to reason using, for example, the direction or slope of each line. This is a numerical reasoning process.

In this section, students combine the graphic method to find a point of intersection with the use of equations. In the first problem in this section, they estimate the coordinates of the point of intersection graphically. Since the graph is not easy to read in detail, there is a need to check whether the point of intersection they found is indeed correct. Students, therefore, check the coordinates by using the equations. The equations here are just used for a check; students can fill in the coordinates for $x$ and $y$ and see if the equations result in true statements.

A graph of an equation like $y = 3x + 5$ is a representation of all the $(x, y)$ points that are solutions to the equation. Therefore, when two lines are graphed, their point of intersection is the only point that solves both equations. By linking the lines in the graph to their equations using arrows, the method for solving frog problems by algebraically solving the equation from Section D, is related to finding the point of intersection of two lines. When solving the equation, students need to find the $x$-coordinate of the point of intersection as well as the $y$-coordinate.

In the rest of the section, students use any method they like for finding points of intersection and solving equations. They connect the graphic and algebraic method explicitly for a deeper understanding of the two. Students also explore the relationship between parallel lines and graphs of lines without intersection points in this section.

### Learning Outcomes

Students will be able to find and use equations for lines in the slope and $y$-intercept form. They can graph this type of equations in a coordinate plane as lines and are able to find the intersection point of two lines algebraically as well as graphically. Students understand the similarities between these two strategies. They can choose an appropriate way to solve equations.