Overview for Families

Mathematics in Context unit: ***Figuring All the Angles***

Mathematical strand: **Geometry and Measurement**

The following pages will help you to understand the mathematics that your child is currently studying as well as the type of problems (s)he will solve in this unit.

Each page is divided into three parts:

- *Section Focus*
  Identifies the mathematical content of each section.

- *Learning Lines*
  Describes the mathematical flow of each section.

- *Learning Outcomes*
  Outlines what students should know and be able to do at the end of each section.

"From the very beginning of his education, the child should experience the joy of discovery."

Alfred North Whitehead

Figuring All the Angles

Section A A Sense of Direction

Section Focus

Students explore the cardinal directions and compare the relative and absolute use of directions. At the start of this section, students make a sketch of their classroom and indicate where north is. Next they use cardinal directions when looking at the sun's position in the sky, describing places in their local region, and sketching the room where they sleep. They discuss the use of north, east, south, and west to describe regions of the United States. This brings up the difference between relative and absolute directions. The use of directions is extended to the common compass directions northeast, northwest, southwest, and southeast. The instructional focus of Section A is to

- **indicate direction using the cardinal directions, and**
- **use directions to solve simple problems.**

Learning Lines

Main Approach to the Concept of Angles

The main focus of this unit is on angles. However, the way angles are introduced in this unit in many ways differs from a more traditional approach where students encounter angles in geometric shapes and learn how to measure and draw angles. This more static type of situations in which angles appear can be found only at the end of this unit.

The approach that is chosen here is a dynamic one: Angles appear in situations concerning orientation and especially navigation. In order to get from one place to another (whether on the map or in the real world), a certain distance is covered in a certain direction. The direction can be described with cardinal directions or, more precisely, with degrees measured from north. To change from one direction to another, a turn is made, which can again be measured in degrees. And if you look at the track that is made, each turn results in an interior angle, which can also be measured in degrees. As a result of this approach, students will have a rich perspective on the appearance of angles in the real world and on paper.

Introduction of the Compass

Section A is the first step on the road to dynamic angle understanding. It is meant to let students explore their notions of direction, particularly the direction north. Students may have the idea that "north is up" in the context of maps. After this section, they should realize that there is more to it. On a map you can indicate where north is. But how do you determine where north is? And what is north in fact? What does it have to do with the North Pole? What is a compass, and how does it work?

The compass is a very important tool that will be used throughout this unit, and Section A provides a broad exploration of this tool. The section starts with the four cardinal directions, refined to eight main directions. In Section C, the compass will be refined further to the use of degrees. And from there, it will be only a small step to the circular or polar grid system. This resembles the compass card, an instrument that can be used for angle measurement.

Figuring All the Angles

Section A A Sense of Direction

Learning Outcomes

Students can indicate a direction using cardinal directions (i.e., north, east, south, west, northeast, northwest, southeast, and southwest), both in the real world and on maps. They understand the difference between absolute and relative directions.

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Section B Finding Your Way

Section Focus

Through the context of maps, students investigate the rectangular grid system. Students draw parallel and perpendicular streets on the map of a growing small town. They compare systems for naming streets. They locate places and buildings on maps and navigate on the grid-like map using distances and cardinal directions. Distances on the map are measured "as the crow flies" and using taxicab distances. The instructional focus of Section B is to

- **identify position on a rectangular grid;**
- **use the scale on a map to determine distance; and**
- **follow and give written directions.**

Learning Lines

Rectangular Grid System

There are different ways to locate places. One is using a polar coordinate system, for which the previous section offered an introduction to the underlying concepts. This will be further investigated in Section D. The other way to locate places is using a rectangular grid system, the focus of Section B. The concepts of parallel and perpendicular play an important role. Also, students need to use the scale line on a map to determine distances.

Learning Outcomes

Students understand how positions can be identified using a rectangular grid system. They combine their knowledge of cardinal directions to indicate a direction with the ability to determine distances on a map using the scale line. They understand the difference between distances as "the crow flies" and taxicab distances.

Figuring All the Angles

Section C Directions

Section Focus

Students use a map to estimate directions to different airports. They find that the eight main directions are not precise enough to describe the course of planes. Therefore, the compass directions are further refined to degrees. Students are familiarized with the compass card and learn how to use headings expressed in degrees for navigation purposes. Students again use maps, both to draw possible routes using headings and to follow written heading and distance instructions. The instructional focus of Section C is to

- **indicate directions or headings using 360°, and**
- **solve simple problems involving headings and distances.**

Learning Lines

The Compass Card

The compass card is a tool used throughout *Mathematics in Context* both to navigate and measure angles. The compass card is a refinement of the main directions students investigated in Section A. Degrees divide a circle in 360 parts. North is 0°, east is 90°, south is 180°, and west is 270°. The use of 360° makes it possible to plot courses precisely, if used in combination with distances. In fact, this navigation process implies the informal introduction of vectors (distance combined with direction). It is important that students learn how to use the compass card appropriately following the instructions described in the student book, since this is crucial for the problems that will be solved in the following sections.

A protractor, which can be used for measuring angles, will be introduced in Section G. This tool can only be applied for measuring angles, not for navigation purposes.

Learning Outcomes

Students understand the ideas underlying the compass card and know how to use it to measure and draw headings. They understand that using headings in combination with distances makes it possible to plot courses.

Figuring All the Angles

Section D Navigation and Orientation

Section Focus

The circular or polar grid system is introduced in the context of a radar screen used by an air traffic controller. Planes are located using a compass direction expressed in degrees and a distance expressed in kilometers or miles. Students learn about the differences between the rectangular grid they used earlier and the polar grid. The instructional focus of Section D is to

- **identify positions using a polar grid and**
- **compare rectangular and polar grids.**

Learning Lines

Polar Grid System

A polar grid is similar to a compass card, but in addition it has evenly spaced concentric circles for measuring distance. Polar grids are used to locate points in terms of their direction and distance from a fixed central point. In the context of air traffic control, the center represents the airport, and a plane located at the position 70°/25 miles is in the direction 70° from the airport at a distance of 25 miles. The polar grid can also be used to describe movement from one location to another using degrees to indicate the direction of the movement and the changing distance measured from the central point.

Building on the idea of the polar grid and the use of the compass card, students find another way to locate a point on a map. Given the location of two fixed points, a third location can be identified if the directions from each of the two fixed points to the third point are known: The third point is where the two lines indicating the directions intersect.

Learning Outcomes

Students understand how positions can be identified using a circular or polar grid system. They are able to describe the differences between the rectangular and the polar grid systems. Also, they can use the compass card to plot given directions from fixed points in order to determine an unknown location.

Figuring All the Angles

Section E Changing Directions: Turns

Section Focus

Students explore changes in direction. The context is airplanes: The conversation between the pilot and the air traffic controller shows how the pilot is instructed to change the plane's heading from one to another. Students learn how to represent the conversation in a drawing. The change of direction or heading is called a *turn*. Through an activity, students simulate air traffic control by giving headings and making turns. The dynamic representation of an angle is reinforced by having students act out the landing of an airplane, with the traffic controller instructing the pilot to change headings before arriving at the airport. Students will learn to compute the total turn after a sequence of given turns, both to the left and right. The instructional focus of Section E is to

• define *turn as* a change from one direction (heading) to another.

Learning Lines

Turn

So far in this unit, degrees have been used only in the context of directions or headings. The word *turn*, defined as a change in direction, is introduced in this section. A turn can be described using degrees. In fact, what we are dealing with here is an angle that can be measured. The word *turn* is used since it is tied to the dynamic airplane context. The term *angle* should not be used yet. It is used in the next section to describe the resulting angles (called interior angles) in the tracks of a dogsled. Terminology use needs to be carefully controlled so that students do not become confused.

In this section, much attention is paid to stepwise learning to make precise representations of a series of turns as changes of direction or heading. Turns can be made to the left and to the right. The present and new headings are represented with arrows; the degrees are measured relative to north. Students again need the compass card and a ruler to construct these drawings.

Learning Outcomes

Students understand that a turn is a change of direction or heading. They are able to represent a sequence of turns (left or right) in a drawing with clear labels of old and new headings. They can determine how many degrees the total turn is after a sequence of turns, by means of both drawing and computation.

Figuring All the Angles

Section F From Turns to Angles

Section Focus

The context of a sled in the snow was chosen as the start for this section since the course that was driven is visible in the snow tracks. Students investigate the angles visible in the tracks. These angles (called interior angles) are the result of changes in the heading or direction (turns). In this section, formal terminology and notation for angles is introduced, which is applied by students when drawing angles. Through the study of regular polygons, students investigate the relationship between turns and interior angles and learn about the sum of interior angles of regular polygons. The instructional focus of Section F is to

- **define the relationship between a turn and the resulting angle, called interior angle, and**
- **investigate turns and interior angles of regular polygons.**

Learning Lines

Interior Angles

This section offers a change of students' perspective. Their role as actor (pilot, captain, or sled driver), setting courses and making turns to change directions, changes to that of spectator, observing a path created by the turns of a plane, boat, or sled. The angles that are visible in the track are defined as interior angles. Students are meant to discover that the turn and the interior angle add up to 180°.

If, after making a sequence of turns, the end point is the same as the starting point, it means that the turns add up to a total of 360°. In the case of regular polygons the measure of each turn (and as a consequence of each interior angle) is the same. For example, for an octagon, each turn measures 45° since $8 \times 45° = 360°$. Each interior angle measures 135° (180° − 45°).

Learning Outcomes

Students understand the relationship between turns and interior angles. Given the measure of the turn they can figure out the measure of the interior angle. Students can use formal terminology and notation concerning angles. Students are able to describe regular polygons in terms of the turns needed to create them and the measure of their interior angles.

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Section G Angles and Their Measures

Section Focus

After identifying objects in the real world showing 90º angles, in this final section students learn to classify angles: right, acute, obtuse, and the 180º angle as a straight line. Some other formal terminology is introduced: the vertex and the sides of an angle. Besides the compass card, students learn how to use a protractor to measure angles.

The context is geometric shapes, whether in real-life objects, such as mosaic tiles and starfish, or in plain geometric figures. Students learn how the size of an angle can be determined by measuring, estimating, or reasoning. The instructional focus of Section G is to

- **introduce right, acute, and obtuse angles;**
- **measure and estimate angles in geometric shapes;**
- **find the size of an angle in a geometric shape by reasoning; and**
- **use a protractor as well as a compass card to draw and measure angles.**

Learning Lines

Angles and Shapes

The more static approach of angles is the subject of this final section. The approach in the previous sections was a dynamic one: angles as they appear in situations involving navigation. Students learned that turn is a change of direction (indicated through cardinal directions or, more refined, in degrees) that can be measured in degrees. A turn results in an angle in the track, called interior angle, which can also be measured in degrees. Now that students are familiar with the broad concept of angles, they are ready for formal terminology, classification, measuring, and reasoning about angles as they appear in the context of more formal geometry.

At the end of the section, students informally learn the rules for the sum of angle measurements in a triangle. This will be formalized in the unit *Triangles and Beyond*.

Learning Outcomes

Students can discern different types of angles: right, acute, obtuse, and straight angles. They know formal notation and the terms *vertex* and *side*. Students can measure, estimate, draw, and reason about the size of angles, both using a compass card and a protractor.