Overview for Families

Mathematics in Context unit: ***Facts and Factors***

Mathematical strand: **Number**

The following pages will help you to understand the mathematics that your child is currently studying as well as the type of problems (s)he will solve in this unit.

Each page is divided into three parts:

- *Section Focus*
  Identifies the mathematical content of each section.

- *Learning Lines*
  Describes the mathematical flow of each section.

- *Learning Outcomes*
  Outlines what students should know and be able to do at the end of each section.

"From the very beginning of his education, the child should experience the joy of discovery."

Alfred North Whitehead

Facts and Factors

Section A Base Ten

Section Focus

The focus of this section is to develop an understanding of the base-ten number system, to review and develop the notation and naming of large numbers, and to introduce the students to *exponential notation* and *scientific notation*.

Learning Lines

Number Systems and Number Sense

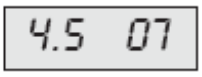
Students will use and further develop their number sense and computational skills when they study place value and the notation and naming of large numbers. When students study the Egyptian number system and compare it with our (Arabic) number system, they will notice one important difference: Our system is based on place value. Multiplying a number by ten actually means that the value of each digit changes to a greater value, and, thus, each digit changes position.

Exponential Notation

Large numbers give cause for the introduction of powers of ten: $1{,}000{,}000{,}000 = 10^9$. The concepts of base and exponent are made explicit. In Sections B, D, and E, students will investigate powers with other bases.

Scientific Notation

Students investigate how a calculator displays very large numbers and makes connections between the product of a number and a power of ten—that is, scientific notation. Students learn how to interpret large numbers shown on a calculator's display. For example:



4.5 07 is 4.5×10^7, or 45,000,000, which is 45 million.

Learning Outcomes

Students have further developed their number sense and have a conceptual understanding of the base-ten number system, large numbers, and powers of ten. They are able to interpret a large number shown on a calculator as a product of a number and a positive power of ten and are able to write this product as a single number.

Facts and Factors

Section B Factors

Section Focus

The context of pixels of digital images is a vehicle to touch upon several mathematical concepts and to contribute to students' development of number sense. Students revisit the bar model, find the total number of pixels of a square picture, investigate patterns in a table, revisit the use of arithmetic trees, and investigate divisibility rules. They read and draw graphs, find factors of a number, and are introduced to the concept of prime numbers.

Learning Lines

Models

The download bar may remind students of a model they learned to use in the sixth-grade number units: the bar model.

Squaring Numbers

When students find the total number of pixels of a square picture, with 12 pixels along each side, they calculate 12×12, which can be written as 12^2, or 144. Numbers like 144 that result from squaring a number are called *square numbers* or *perfect square numbers*. In Section D, squaring numbers is revisited and related to the inverse operation: "unsquaring" or taking the square root.

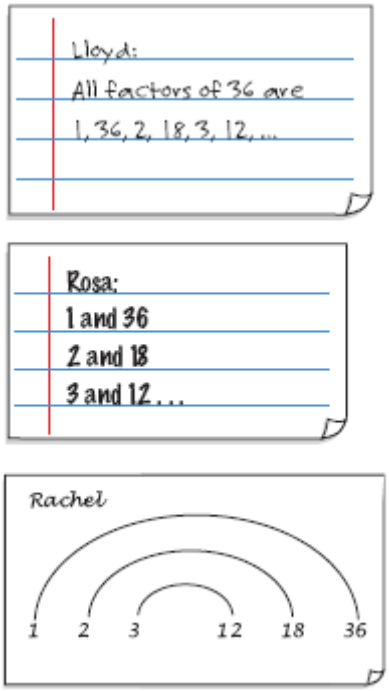
Factors

Students find factors of a number supported by visualizing all possible rectangular arrangements of a fixed number of square pictures. Students use graphs to represent factors of 23, 24, and 25. They determine for which numbers the graph of points representing factors would have an odd number of points (perfect square numbers) and only two points (prime numbers). In this way, they further explore the concept of a factor and of prime numbers and further explore the concept of perfect square numbers.

Facts and Factors

Section B Factors

Students see three different strategies for finding the factors of a number:



In Section C, upside-down arithmetic trees (factor trees) are used to find the prime factorization of numbers.

Learning Outcomes

Students are introduced to the concepts of (perfect) square numbers, factors, and prime numbers. They are able to find all factors of a number. They are able to use divisibility rules to identify numbers that are divisible by 2, 3, 5, or 9.

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Section C Prime Numbers

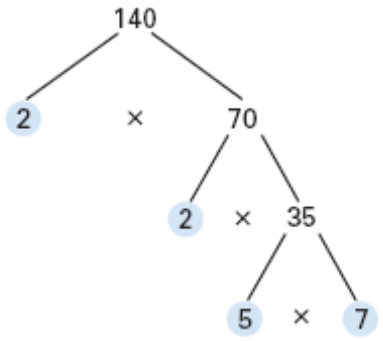
Section Focus

The focus of this section is to develop students' understanding of the concepts of factors, prime numbers, and composite numbers. Students use upside-down arithmetic trees (factor trees) to factor a number into a product of primes. They learn and compare a second method to factorize numbers. Students are introduced to the sieve of Eratosthenes and use this method to find the prime numbers between 1 and 100.

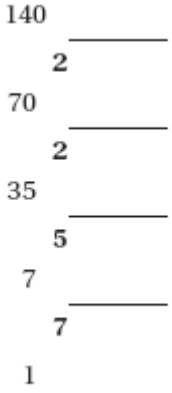
Learning Lines

Algorithms

In this section, students continue to use arithmetic trees from Section B to help organize their calculations, and they review the order of operations. Then the arithmetic trees are inverted and used to factor a number into a product of primes. The upside-down arithmetic trees are called *factor trees*.



Students learn a second algorithm to find the prime factorization of a number:



Every number can be factored into a unique product of prime numbers. For example, the number 42 can be written as $2 \times 3 \times 7$, while the number 140 can be written as $2 \times 2 \times 5 \times 7$, or $2^2 \times 5 \times 7$. In Section E, students are explicitly asked to write a prime factorization as a product of powers. The concept of algorithm is not discussed here. It is introduced in the unit *Revisiting Number*.

Facts and Factors

Section C Prime Numbers

Factors, Prime Numbers, and Composite Numbers

In Section B, students were introduced to the concept of prime numbers. In this section, students will further explore prime numbers and learn the concept of composite numbers. Students also learn to distinguish between factors, multiples, and prime numbers. For example, the number 6 has the factors 1, 2, 3, and 6 because each number divides evenly into 6. Six is a multiple of 2 because the multiples of 2 are 2, 4, 6, 8, and so on. Six is also a multiple of 3 because the multiples of 3 are 3, 6, 9, 12, and so on. Prime numbers (such as 2, 3, 5, 7, 11, and so on) are numbers that have exactly two factors, 1 and themselves. Composite numbers (such as 4, 6, 8, and so on) are numbers greater than one that have more than two factors. Two is the only even prime number.

Learning Outcomes

Students have developed an understanding of prime and composite numbers. They are able to use a strategy to completely factor a number into a product of primes.

Facts and Factors

Section D Square and Unsquare

Section Focus

In the context of area, students revisit squaring numbers from Section B. They square and "unsquare" numbers, and they are formally introduced to square roots and the $\sqrt{\ }$ symbol. Students square simple mixed fractions, such as 1 1/2 $\times$ 1 1/2 and (4 1/2)^2, using drawings of squares. Then they learn how to use area and drawings of rectangles as a model for multiplying mixed fractions, such as 4 1/2 $\times$ 2 1/2.

Learning Lines

This section formally introduces the concepts of square and square root, with special attention to the properties of squares and of square roots as well as to the mathematical notation.

Operations

In the unit *Expressions and Formulas*, students investigated operations and reversed operations using arrow language. Squaring and taking the square root (unsquaring) are also inverse operations. Arrow language can be used to visualize this, especially when used in the context of area:

Area of a square $\underset{x^2}{\overset{\sqrt{\ }}{\rightleftarrows}}$ side length

Models, Operations with Fractions

In this section, the operations with fractions that were not formalized in the unit *Fraction Times* are revisited, especially multiplying mixed numbers. The area model is developed and used to develop students' understanding how to multiply fractions and mixed numbers.

Number

Some square roots are integers or whole numbers, for example, $\sqrt{25} = 5$.

Other square roots can be written as a fraction or decimal, for example, $\sqrt{\frac{1}{4}} = \frac{1}{2}$ or $\sqrt{6.25} = 2.5$.

A third kind of square root cannot be written as a terminating or repeating decimal or a fraction, such as $\sqrt{52}$. The numbers in this third category are called *irrational numbers*. The distinction between integers, rational numbers, and real numbers need not be made explicit to students. Students do not need to know the formal mathematics yet. The different kinds of number sets are revisited and formalized in the eighth-grade unit *Revisiting Numbers*.

Learning Outcomes

Students have developed an understanding of squaring and square root (unsquare). They have learned the formal notation for square root and use calculators to find square roots of numbers. They have developed and are able to use the area model to calculate a fraction of a fraction and to multiply simple mixed numbers.

Facts and Factors

Section E More Powers

Section Focus

Students further explore exponential notation using powers of different bases. They solve problems where powers are involved and start to discover and use calculation rules, which are formalized later in the unit *Revisiting Numbers*. Students review prime factorization from Section D and use exponential notation to write each factorization as a product of powers. Students are introduced to the binary number system; this is in contrast to the decimal number system they reviewed in Section A.

Learning Lines

Exponential Notation

In the beginning of this unit, the powers of ten were introduced. In Sections B and D, square numbers were written as a power with two as exponent. In this last section, students use exponential notation for powers with other bases. Once students learn how to express numbers in exponential notation, they perform multiplications of powers with the same bases by using calculation rules without referring to the meaning of each number expressed in exponential notation. For example, to calculate $3^2 \times 3^5$, students focus on adding the two exponents (2 and 5) to find the product, 3^7. Rules for calculations with powers are made explicit in the unit *Revisiting Numbers*.

Algorithm

In this section, students are introduced to the Egyptian doubling method. This algorithm is based on powers of two: Each natural number can be written as a sum of powers of two. For example 13 × 51:

$$\mathbf{13} \times 51 = \mathbf{1} \times 51 + \mathbf{4} \times 51 + \mathbf{8} \times 51$$
$$13 = 1 + 4 + 8 = 2^0 + 2^2 + 2^3$$

Students compare this method with a method that uses a ratio table.

1	2	4	8	13
51	102	204	408	663

Double each time, and then add columns 1, 2, and 4.

Number Systems and Number Sense

While the decimal number system is based on the use of powers with base ten and the digits 0 through 9, the binary number system uses powers with bases two and the digits 0 and 1. Both number systems are based on place value. In the decimal number system, powers of ten can be used to expand a number, for example, $2{,}054 = 2 \times 10^3 + 0 \times 10^2 + 5 \times 10^1 + 4 \times 10^0$. In the binary number system, powers of two can be used to expand a number, for example, $1{,}001 = 1 \times 2^3 + 0 \times 2^2 + 0 \times 2^1 + 1 \times 2^0$. This expansion can be used when we want to convert a number from the binary number system to the decimal number system: $1{,}001 = 1 \times 2^3 + 0 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 = 8 + 0 + 0 + 1 = 9$.

Facts and Factors

Section E More Powers

Learning Outcomes

Students have further developed their number sense and have developed a conceptual understanding of exponential notation. They are able to use a strategy to factor a number into a product of primes and use exponential notation when applicable. Students understand and use informal rules for operations with powers. They also understand the binary number system.