Overview for Families

Mathematics in Context unit: ***Expressions and Formulas***

Mathematical strand: **Algebra**

The following pages will help you to understand the mathematics that your child is currently studying as well as the type of problems (s)he will solve in this unit.

Each page is divided into three parts:

- *Section Focus*
  Identifies the mathematical content of each section.

- *Learning Lines*
  Describes the mathematical flow of each section.

- *Learning Outcomes*
  Outlines what students should know and be able to do at the end of each section.

"From the very beginning of his education, the child should experience the joy of discovery."

Alfred North Whitehead

Expressions and Formulas

Section A Arrow Language

Section Focus

This section introduces the use of arrow language as a way to record a series of operations. Students use arrow language to help make calculations in different situations. Toward the end of the section, students begin to use arrow language to look at shortcuts in making "smart" calculations. The instructional focus of Section A is to

- **describe and perform a series of calculations using an arrow string;**
- **rewrite numerical expressions to facilitate calculation;* and**
- **interpret relationships displayed in tables.***

***These goals are introduced in this section and assessed in other sections in this unit.**

Learning Lines

Arrow Language

Arrow language is introduced as a way to represent and perform a series of calculations involving addition and subtraction. Arrow language is a notation for recording results after each step in a series of calculations. It shows intermediate results within a series of arithmetic operations. Using arrow language, calculations can be written in the order they appear without the risk of misusing the equal sign within a series of calculations.

The below way of recording calculations creates a string of untrue statements. For example, 10 + 6 does not equal 16 + 3.

$$10 + 6 = 16 + 3 = 19 + 3 = 22 - 4 = 18$$

Arrow language here is used to keep track of changes in numbers or quantities. Each change is represented by an arrow.

$$10 \xrightarrow{+\,6} 16 \xrightarrow{+\,3} 19 \xrightarrow{+\,3} 22 \xrightarrow{-\,4} 18$$

$$\text{starting number} \xrightarrow{\text{action}} \text{resulting number}$$

Students need to consider that the order of operations is not taken into account using arrow language. This is not addressed formally in this section. This will be done later in the unit.

Other Representations

Information on changing numbers and quantities is often represented in a table. Students can interpret the information from such a table and use it to make calculations solving a problem; they can also complete the table with the missing information.

Learning Outcomes

Students can use arrow language to represent and perform calculations involving addition and subtraction. Students can extract information presented in a story as well as information presented in tabular form to make the appropriate calculations needed to solve a problem using arrow language. Students know that connecting series of calculations with equal signs can be wrong; they also know that the use of arrow language helps to avoid these difficulties.

Expressions and Formulas

Section B Smart Calculations

Section Focus

Students use arrow strings to make calculations in a context situation as well as calculations outside of a context. Students continue using only addition and subtraction and identify strategies to simplify calculations represented by arrow strings and by rewriting them. Students improve their number sense by solving the same problem with different arrow strings and rewriting calculations with "friendly numbers" to facilitate computation. In some cases, students will shorten arrow strings, and in other cases, they will lengthen them. Students determine when it is easier to use a shortened calculation and when it is easier to use a lengthened one. The instructional focus of Section B is to

- **describe and perform a series of calculations using an arrow string and**
- **rewrite numerical expressions to facilitate calculation.**

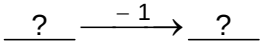
Learning Lines

Arrow Language

Students know from the previous section how arrow language can be used to record and perform a series of calculations in the order they appear. The focus in this section is on rewriting arrow strings. Students discover how a series of calculations represented by an arrow string can be summarized as a one-step calculation with one arrow. The focus in this section shifts to finding strategies to make computations involving arrow language easier. This can be done either by shortening arrow strings or by lengthening them. "Friendly numbers," like 100 or other multiples of 10, are used in the easier strings. The easier calculations can often be done mentally.

Arrow Strings and Variables

In this section, the use of variables is informally introduced, connected to the use of arrow strings. This is done by using a question mark. First a question mark is used to represent an unknown. Later the question mark, or any other symbol students choose, is used to represent "any" number. Use one arrow to show the result for any first number.



The use of arrow language to represent formulas and procedures is revisited and formalized in Section C.

Number Sense

Students use and develop number sense in this section. The context of money and making change helps students make estimates and perform calculations with decimals. Students both use and extend their number sense when they learn about skillful ways to break calculations into parts (lengthen arrow strings) or to combine calculations into easier ones (shorten arrow strings). To do so, they need to rewrite or rethink a given number, such as expressing 99 as 100 – 1 or 104 as 100 + 4. Sometimes combining several numbers into one that is easier to use, such as combining 31 and 19 to make 50, facilitates computation. These number sense strategies are effective for helping students become proficient at mental computation and can be used as tools to verify work done by their calculators.

Expressions and Formulas

Section B Smart Calculations

Learning Outcomes

Students can use strategies to make arrow string calculations easier. They use number sense to rewrite arrow strings into ones involving easier calculations. Students can mentally perform the easier calculations. Students know when to shorten and when to lengthen arrow strings in order to make calculations easier to perform.

Expressions and Formulas

Section C Formulas

Section Focus

Students use arrow language to describe calculations for a variety of contexts. Besides addition and subtraction, now multiplication and division are also used. Arrow string formulas are introduced to describe the calculation in more general terms. Students also use arrow strings to analyze solutions and formulas and begin to develop techniques for working backwards. Students also compare two formulas to decide whether or not they are equivalent. Informally, students use reverse arrow strings to find a missing value in a formula. Students combine the use of (arrow string) formulas with the use of tables and graphs to solve a problem. The instructional focus of Section C is to

- **use word variables to describe a formula or procedure;**
- **use and interpret simple formulas;**
- **interpret relationships displayed in formulas, tables, and graphs;**
- **use reverse operations to find the input for a given output; and**
- **use formulas in any representation (e.g., arrow language, arithmetic tree, words) to solve problems.**

Learning Lines

Arrow Strings

Students make arrow strings fitting problem situations, and they perform calculations with these strings. It is important that students have some understanding of the input and corresponding output values with the context of each problem to determine whether or not their calculated answers are reasonable. The focus is not only on finding the output of the string, but also on finding the input that results in a given output.

Arrow strings that are used repeatedly to answer questions, such as the price of an order of fruit or the number of cups that can be stacked, are generalized into arrow string formulas.

$$\text{weight} \xrightarrow{\times \$1.50} \text{price}$$

$$\text{number of cups} \xrightarrow{?} \_\_?\_\_ \xrightarrow{?} \text{height of stack}$$

Learning Outcomes

Students can create and use arrow string formulas to describe situations and to solve problems. Students can make calculations with arrow strings, using the four basic operations. Students can translate between arrow string formulas and common formulas with an equal sign. Students can make a table and a graph for a given formula using arrow strings.

Expressions and Formulas

Section D Reverse Operations

Section Focus

In this section, students gain experience in rewriting and reversing arrow strings. Students reverse arrow strings for calculations with numbers as well as arrow string formulas. Reversing arrow strings is an informal strategy for solving equations. The instructional focus of Section D is to

• use and interpret simple formulas;
• use reverse operations to find the input for a given output;
• use word variables to describe a formula or procedure;*
• solve problems using the relationship between a mathematical procedure and its inverse; and
• use formulas in any representation (e.g., arrow language, arithmetic tree, words) to solve problems.*

***These goals are addressed in this section and assessed in other sections in the unit.**

Learning Lines

Reversing Arrow Strings

As in the previous section, all four operations (i.e., addition, subtraction, multiplication, and division) are used and combined in arrow strings. Students learn to make "calculations backwards" to find the input of a calculation when the output is given. This "going backwards" is first addressed in the context of converting distances. In this context, it makes sense to be able to go forwards and backwards, that is, from miles to kilometers and from kilometers to miles.

Later students reverse a calculation arrow string, and this reversing process is more formally addressed. (See problems on student book page 28.) Reversing arrow strings for calculations as well as arrow string formulas is a preliminary step in preparing students to later solve algebraic equations with one variable. Students use the idea that addition and subtraction are inverse operations, as are multiplication and division. They begin to see that addition "undoes" subtraction and vice versa. Of course, the same principle holds true for multiplication and division. This understanding of inverse operations helps lay the foundation for solving equations.

Number Sense

In this section, students use and develop number sense when they think about an easier way to convert distances, mentally avoiding decimal numbers. They rewrite arrow string formulas and compare how reasonable an estimate the output of the different strings will give.

Learning Outcomes

Students will be able to use reverse operations to find the input for a given output, and they can solve problems using the relationship between a mathematical procedure and its inverse.

Expressions and Formulas

Section E Order of Operations

Section Focus

Students learn to investigate situations in which the order of operations is an issue. The order of operations can be obvious from the context, but this is not always the case. Students are introduced to arithmetic trees and how to use this model. They also investigate how arithmetic trees can be used with formulas. The instructional focus of Section E is to

• use formulas in any representation (e.g., arrow language, arithmetic tree, words) to solve problems;
• describe and perform a series of calculations using arrow strings and arithmetic trees;
• interpret relationships displayed in tables;
• use conventional rules and grouping symbols to perform a sequence of calculations;
• rewrite numerical expressions to facilitate calculation;
• use word variables to describe a formula or procedure; and
• generalize from patterns to symbolic relationships.

Learning Lines

Arrow Strings and the Order of Operations

Students write and use arrow strings to make calculations. They notice that if they want to write the arrow string calculation as an expression, there is a need to make agreements on the order of operations. In most real-life problems, the context of the situation helps to determine the order of the calculations. The order of operations in an arrow string is immediately clear but cannot be translated directly into a numerical expression.

Although the history of the order of operations is unclear, it is a standard mathematical convention. The order of operations is as follows:

• if grouping symbols are used, perform the operations within the grouping symbols first;
• perform all multiplication and division in order from left to right; and
• then perform all addition and subtraction in order from left to right.

Learning Outcomes

Students will be able to describe and perform a series of calculations using an arithmetic tree. From different possible trees, they can choose the one for which mental computation is easiest. To do so, they use number sense. Students can also write a formula, which cannot be written as an arrow language formula, as an arithmetic tree formula in which the order of operations is shown. They can use formulas in any representation (e.g., arrow language, arithmetic tree, words) to solve problems. Students know the standard order of operations, and they know how parentheses can be used to get the order of operations required for a specific situation.