

Overview for Families

Mathematics in Context unit: **Comparing Quantities**

Mathematical strand: **Algebra**

The following pages will help you to understand the mathematics that your child is currently studying as well as the type of problems (s)he will solve in this unit.

Each page is divided into three parts:

- *Section Focus*
Identifies the mathematical content of each section.
- *Learning Lines*
Describes the mathematical flow of each section.
- *Learning Outcomes*
Outlines what students should know and be able to do at the end of each section.

“From the very beginning of his education, the child should experience the joy of discovery.”

Alfred North Whitehead

Comparing Quantities

Section A Compare and Exchange

Section Focus

In this section, students exchange goods and quantities to solve problems involving bartering, balances, and simple equations. In this way, they informally solve problems involving systems of equations.

Learning Lines

Fair Exchange

In this section, students use their informal understanding of bartering and exchanging items to solve equation-like problems. Within the context of bartering, students are introduced to the concept of substitution by the method of fair exchange. Throughout this unit, students will use these exchange situations to develop more sophisticated strategies for solving equation-like problems. In formal algebra, the exchange process is called *substitution*.

Equations

In this section, students begin to solve equation-like problems. In formal algebra, these equation-like problems are referred to as *a system of equations*. For instance, students are shown two combinations of fruits that balance a scale, and they must exchange, add, and subtract to find what will balance with a single piece of fruit. Exchange or balance situations like this represent equivalent quantities. The balance represents an equal sign. The total weight of the items on the left side equals the total weight of the items on the right side.

Variable

In this section, the variables are presented in pictures. Students are encouraged to use words or symbols to record and explain the process used to solve the problems. The problems in this section involve the number of items rather than the monetary value of the items. In later sections, students find the cost or weight of each item and the cost or weight of a combination of items.

Learning Outcomes

Students use their informal knowledge of bartering and exchanging to solve problems. Students develop their reasoning skills. They informally solve systems of equations. Students use pictures, words, or symbols to describe one's own solution process, and they are able to follow the solution process of someone else.

Comparing Quantities

Section B Looking at Combinations

Section Focus

This section introduces combination charts to organize information about combinations of two items. Students explore exchange patterns in the combination charts and use the patterns to solve problems.

Learning Lines

Combination Chart

This section uses a combination chart as a means for investigating systems of equations with two unknowns. For instance, this chart is used to answer questions about combinations of two different cabinet sizes that can be put along the wall of a room. Mathematically spoken, the general equation, $Ax + By = C$, is portrayed in the combination chart. In the case of the cabinets, A and B represent the number of long and short cabinets, and the variables x and y stand for the two different widths, 60 cm and 45 cm. C stands for the value of the combination, or the total length of the combination.

Fair Exchange

Students use the chart to show that exchanging three Long Cabinets for four Short Cabinets doesn't influence the total length.

Systems of Equations

In most of the problem situations of this section, students know the value of each item and find the value of the combination. Later in this unit, students use the values of the combinations to find the value of the items. This involves solving systems of equations.

Learning Outcomes

Students are able to interpret and use combination charts to discover, investigate, and extend patterns and to solve problems. Students have further developed reasoning skills to solve equation-like problems. They are able to interpret a mathematical solution in terms of the problem situation.

Comparing Quantities

Section C Finding Prices

Section Focus

This section focuses on exchange strategies to determine the price of individual items in a purchase. Students also use combination charts to solve equation-like problems.

Learning Lines

In the previous section, students used the value of one item of each kind to find the value of a combination. In this section, students use the value of the combination of items to find the value of one item of each kind. Students create new combinations from given ones by adding, subtracting, or extending patterns.

Systems of Equations

In each context problem in this section, students are given two linear equations (in picture or story form) with two unknowns, which are the prices of the two items. For instance, students are given the total price of two umbrellas and a cap and the total price of two caps and an umbrella and must find the price of one cap and one umbrella.

Fair Exchange

To solve this sample problem, students can use exchange strategies: Compared to the first picture, in the second one there is one pair of sunglasses less but two pairs of shorts more. Note that the total price is the same for both combinations. Thus, one pair of sunglasses can be exchanged for two pairs of shorts. When in the last picture one pair of sunglasses is exchanged for two pairs of shorts, the result will be five pairs of shorts for a total price of \$50.

Combination Chart

As students relate these pictures to patterns in a combination chart, they discover how patterns in the chart are related to patterns in a succession of equations, like the pattern in the picture equations of the glasses and shorts.

Learning Outcomes

Students have further developed reasoning skills to solve equation-like problems. Students use exchanging, substituting, a combination chart, or other strategies to solve a system of equations, which is presented in pictures or in words. Students create new combinations from given ones by adding, subtracting, or extending patterns. They are able to interpret a mathematical solution in terms of the problem situation.

Comparing Quantities

Section D Notebook Notation

Section Focus

This section introduces the notebook notation to solve problems involving the cost of combinations of three or more different items. Students manipulate rows of numbers in various notebooks in order to isolate the price of an item or come up with a requested combination.

Learning Lines

Notebook Notation

The notebook notation can be used to find the values of three unknowns. The format is like a matrix. In a notebook notation, each number of the three items is shown in a column, and the total price is shown in the last column. Each row in the notebook represents an equation that corresponds to a combination. A row can be multiplied or divided by a constant to form an equivalent equation. Two rows can be added or subtracted to form a new row. By careful manipulations, students can obtain a row that contains only one unknown.

Systems of Equations

Unlike the combination chart, notebook notation can be used to solve problems involving combinations of more than two kinds of items. In addition, notebook notation can be used to record the new combinations. Students should come to appreciate the advantages of notebook notation over guess-and-check strategies and combination charts.

Learning Outcomes

Students will be able to organize information from problem situations using the notebook notation. They interpret and use notebook notation to generate new combinations (equations) and to solve systems of two or more equations given in pictures or words. They are able to interpret a mathematical solution in terms of the problem situation. They informally solve problems that involve systems of equations.

Comparing Quantities

Section E Equations

Section Focus

Students write equations to represent combinations of quantities. They transfer equations into notebook notation and combination charts to solve problems.

Learning Lines

Variables

The use of letters to represent the value of an item, such as weight, length, cost, or number of people, is introduced in this section.

Models and Strategies

In this section, the models and strategies introduced and used in earlier sections (i.e., the method of fair exchange, the combination chart, and notebook notation) are related to equations with variables.

Systems of Equations

For a system of equations to have a unique solution, it must have the same number of equations as variables and all equations must be independent. Dependency occurs when the equations given are multiples of each other and no given equation presents new information. For example:

$$2L + 3M = 5$$

$$4L + 6M = 10$$

These two equations are dependent: The second equation is double the first. They present the same information. If you were to graph these equations, the points solving them would lie on a single line.

Learning Outcomes

Students have some understanding of the concepts of variable and equation. Students are able to produce equivalent equations using pictures, words, symbols, or letters. Students use different representations like pictures, notebooks, charts, and equations and choose strategies appropriate for the problem situation.