Overview for Families

Mathematics in Context unit: ***Building Formulas***

Mathematical strand: **Algebra**

The following pages will help you to understand the mathematics that your child is currently studying as well as the type of problems (s)he will solve in this unit.

Each page is divided into three parts:

- *Section Focus*
  Identifies the mathematical content of each section.

- *Learning Lines*
  Describes the mathematical flow of each section.

- *Learning Outcomes*
  Outlines what students should know and be able to do at the end of each section.

"From the very beginning of his education, the child should experience the joy of discovery."

Alfred North Whitehead

Building Formulas

Section A Patterns

Section Focus

Section A reviews the use of words, tables, NEXT-CURRENT, and direct formulas to represent visual, geometric patterns. Square tiles in two colors form garden paths for the set of a movie. Construction work on a large building used for the movie set involves beams of different lengths. For each pattern in the section, there is an input variable such as *path number* and an output variable that is used to determine a specific feature, such as *number of tiles* for each stage of the pattern. Arrow strings that represent a pattern are used to stress the importance of the order of operations.

Learning Lines

Number Sense

While some students focus on the repetition of geometric patterns, others look at regularities in numbers in the tables they investigate. The order of operations is used in calculations such as $4 + 29 \times 2$. Students need to apply rules of divisibility when investigating whether or not a path exists that requires exactly 100 tiles. (It does not exist because 100 is not divisible by 3.)

Patterns and Regularities

Students investigate and use a variety of different, but mathematically equivalent, direct formulas. For example, $R = 3L + (L - 1)$ is the same as $R = 4L - 1$ because they both describe the same pattern.

At first, students will evaluate these different formulas for different inputs to see they produce the same results. Students will then show that the different formulas represent different geometric or numeric features of the same pattern. Note: At this point, students do not use symbol manipulation to demonstrate the formulas are equivalent. Students compare the advantages and disadvantages of NEXT-CURRRENT and direct formulas to describe a pattern. By repetition of a basic pattern, students are informally introduced to the distributive property. $T = (P + 2) + (P + 2) + (P + 2)$ can also be written as $T = 3 \times (P + 2)$ or $T = 3(P + 2)$.

Models

Arrow strings are used to bridge the gap between a description in words and by a formula. By using arrow strings, a correct order of operations is stressed.

$P \xrightarrow{+\,2} \ldots. \xrightarrow{\times\,4} \ldots. \xrightarrow{-\,4} W$ and $W = 4(P + 2) - 4$ represent the same relationship between patio number (P) and number of white tiles (W).

Learning Outcomes

Students represent a visual pattern in words, symbols, and numbers. They describe geometric patterns using recursive and direct formulas. They recognize the advantages and disadvantages of different representations and use formulas to solve simple problems. Students recognize equivalent representations for the same situation without using symbol manipulation yet.

Building Formulas

Section B Brick Patterns

Section Focus

Students extend and analyze rows of standing and lying bricks that are created from a basic pattern. At first, a string of letters like *SSSLL* is used to describe the repeating part of the pattern. Later, direct formulas are used, for example *Length of Classic* = 8(3*S* + 2*L*), the basic pattern of three standing and two lying bricks used for a brick border, is repeated eight times. The *Length* can be calculated since the length and width of the bricks are given. Students also investigate the effect of switching the position of the bricks in the row on the formula and on the length of the row. Then they determine possible brick patterns from a direct formula without parentheses.

Learning Lines

Number Sense

Students use the dimensions of bricks, given in metric units, to find the length in centimeters of a brick row.

Patterns and Regularities

Students identify a basic repeating pattern from a row of standing and lying bricks. At first, the symbols S and L are used as labels so that students can calculate the number of bricks needed for a row. Then they are used as variables, representing the value of the length and short side of a brick. The mathematical distinction is important but is not made explicit for students at this time; the context supports the meaning of the letters. Students use the distributive property to write direct formulas with and without parentheses. However, the distributive property is not formalized since students may refer to the context. It is not expected that students operate on an abstract level yet. By using the distributive property, students show how formulas that may appear different describe the same pattern.

Models

Strings of letters are used to bridge the gap between a description in words and by a formula.

Learning Outcomes

Students understand how a formula can be helpful if you have to repeat a calculation over and over. They have investigated direct formulas written with and without parentheses, thus exploring equivalent formulas. They are able to find a basic repeating pattern within a given formula.

Building Formulas

Section C Using Formulas

Section Focus

Students use formulas representing linear relationships investigating real-life situations. The relationship between temperatures measured in degrees Celsius or Fahrenheit is explored using a formula for the conversion, a table, and a graph.

$$1.8C + 32 = F \text{ or } F = 1.8C + 32$$

The linear relationship of the two temperature scales is later formalized in the unit *Graphing Equations*.

Students make a model of a staircase and relate rise and tread of the staircase to ease of climbing. The relationship between rise and tread of stairs is studied while using constraints for the rise and the tread.

Learning Lines

Patterns and Regularities

Conversion formulas are reversed to show how formulas describing the same relationship can be rewritten: $F = 1.8C + 32$; $C = 0.56(F - 32)$. Students explore the use of a formula to make estimations mentally and compare this to the formula they used earlier. The steepness of a staircase can also be described as the ratio of the vertical distance to the horizontal distance. This way of looking at steepness is studied in more detail in the unit *Graphing Equations*. In the unit *Looking at an Angle*, steepness is related to the angle and the tangent ratio ($\frac{height}{distance}$).

The linear relationship between the rise and tread becomes very visible in the graph. By using restraints for rise and tread, students are informally introduced to the concepts of *domain* (set of values that have meaning in the formula) and *range*.

Models

Students use a reverse arrow string for converting degrees Celsius to degrees Fahrenheit and use this arrow string to write a formula.

Learning Outcomes

Students have investigated real-life situations using formulas, tables, and graphs. They are able to use formulas converting temperatures, both exact and estimation formulas, and understand why both are useful in different situations. Restraints like $Rise \leq 20$ (cm) are used to graph and to design a staircase that fits the stair building rule.

Building Formulas

Section D Formulas and Geometry

Section Focus

This section reviews and applies formulas students have used in other units, such as *Reallotment*, thus relating different strands in mathematics. Students investigate area and volume in different contexts, such as the area of lichen growing on walls, rocks, and in the tundra and the volume of juice cans and plastic pyramids containing clocks. Square roots and squares are formalized in the unit *Facts and Factors*. This concept is briefly reviewed in this section.

Learning Lines

Number Sense
Students square and unsquare numbers and find square roots of natural numbers. Students learn to use the $\sqrt{\ }$ key on the calculator to find the square root.

Patterns and Regularities
Students look for relationships in a table among the radius of a circle and its area. Different size juice cans are used to strengthen the concept of how the volume of a cylinder changes if the diameter of the circle or the height changes.

Models
In the context of the area of a lichen, students square and unsquare numbers and express this using arrow strings. A common misconception when using a formula like this one: *Volume of a Pyramid* = 1/3 a^2h is explicitly addressed by posing this problem: Matthew made this arrow string:

$$a \xrightarrow{\times \frac{1}{3}} \underline{\quad\quad} \xrightarrow{\text{square}} \underline{\quad\quad} \xrightarrow{\times h} \text{volume}$$

Matthew made a mistake. What was his mistake?

Learning Outcomes

Students recognize the usefulness and restriction of using models (formulas) to investigate some contexts such as in the case of the area of lichen or the volume of a juice can. Students are able to round off an answer to a reasonable number of decimals, according to the situation, and apply their knowledge of square numbers and roots when using formulas for surface area and volume. Students are able to generate and use a table of values and/or graph.

Building Formulas

Section E Problem Solving

Section Focus

In the last section of this unit, students use the knowledge about formulas gained from previous sections to investigate a variety of real-life problems. Students find their normal heart rate by measuring pulse rate and then graph the relationship between maximum heart rate and age. Then they try to find a formula to describe this relationship. Another activity shows a relation between the number of cricket chirps and the air temperature. Students use and investigate a formula that describes this relationship. To resolve this problem, students have to be able to generate and use a table of values and a graph. In the last problem, students have to investigate an archaeological formula for reconstructing damaged Egyptian drawings. These formulas are based on relationships among the heights of various parts of the body.

Learning Lines

Number Sense

Students measure and make such estimations as finding their normal heart rate in beats per minute or looking for the proportion among the heights of various parts of the body. They use estimation rules within a variety of contexts and criticize the mathematical models these are based upon as well as identify limitations of the models.

Patterns and Regularities

In this section, students explore the use of a variety of relationships and formulas. For example, the number of chirps of a certain type of cricket relates to the temperature (in degrees Fahrenheit) of the surroundings.

$$\frac{N}{4} + 40 = F$$

Students should realize this is only an estimate and give reasons for that statement. Is there a maximum number of chirps (N) that could be heard? The answer depends on whether students focus on the chirps the human ear could actually detect per minute or the temperature at which the cricket might survive and still be able to chirp. There should be some understanding of why the relationship is linear. Students' abilities to use formulas, tables, graphs, and explanations in words are evaluated in this section.

Problem Solving

In this section, students are expected to use the mathematical skills and knowledge gained in previous sections to resolve real-life problems. Within different contexts, students have to be creative, to mathematize (extracting the mathematics in a situation and using it to solve a problem), to generalize, and to interpret and organize data.

Learning Outcomes

Students use formulas to solve simple problems, and they appreciate the usefulness of them. Students are able to generate and use a table of values and/or graph. Students are able to recognize the advantages and disadvantages of different representations (i.e., tables, direct formulas, and visual models).