Overview for Families

Mathematics in Context unit: ***Algebra Rules!***

Mathematical strand: **Algebra**

The following pages will help you to understand the mathematics that your child is currently studying as well as the type of problems (s)he will solve in this unit.

Each page is divided into three parts:

- *Section Focus*
  Identifies the mathematical content of each section.

- *Learning Lines*
  Describes the mathematical flow of each section.

- *Learning Outcomes*
  Outlines what students should know and be able to do at the end of each section.

"From the very beginning of his education, the child should experience the joy of discovery."

Alfred North Whitehead

Copyright © by Encyclopaedia Britannica, Inc. All rights reserved.

Algebra Rules!

Section A Operating with Sequences

Section Focus

This section reviews and builds upon the mathematics of arithmetic sequences students have seen earlier, such as in the unit *Patterns and Figures*. The focus of this section is for students to have more practice and flexibility in working with expressions. Extra practice can be found in the *Algebra Tools* resource.

Learning Lines

Expressions to Represent Arithmetic Sequences

For students, it is not obvious at all that different patterns can lead to the same arithmetic sequence. A more formal definition of an arithmetic sequence is provided and discussed in this section. The variable n represents the number of steps; as a consequence, the start number of the sequence corresponds to $n = 0$. Students build expressions, using number strips. They add and subtract expressions and multiply them by an integer or fraction with the help of number strips.

Operations with Expressions

Students add, subtract, and multiply expressions by a constant. Number strips are still used as a model, but students also work with expressions in a formal way. Students operate with expressions horizontally and using stacked notation. An advantage of stacked notation is that students can organize *similar terms* above one another. Number strips are used to review the distributive property, and this property is applied in a more formal manner. Different representations of expressions are compared in order to find equivalent expressions.

Other Representations: The Number Line

The use of a number line helps students understand, for instance, that $n + 2$ is the number that follows $n + 1$ in a sequence of whole numbers, or that $2n - 1$ and $2n + 1$ are consecutive numbers in the sequence of odd numbers. Counting on the number line is yet another way to help students understand the subtraction of two linear expressions, which differ only in their constant term.

Learning Outcomes

Students are able to connect number strips to arithmetic sequences. They build and use expressions that represent the numbers on a number strip. They are able to add, subtract, and multiply expressions in a formal manner (i.e., symbolic manipulation). Students also develop their understanding of how the distributive property can be applied to algebraic expressions.

Copyright © by Encyclopaedia Britannica, Inc. All rights reserved.

Algebra Rules!

Section B Graphs

Section Focus

In this section, the focus is on equations of a straight line. Some of the content from *Graphing Equations* is reviewed here and formalized. For some students, it is not obvious that the different (word) formulas they have seen previously are all part of the same "family" of linear relationships. This meta-relationship between formulas should be emphasized here. Some of the formulas from previous units are reviewed, but teachers may want to show more examples and non-examples as shown at the start of the "Rules and Formulas" section. A graphing calculator can be used in this section. Extra practice can be found in the *Algebra Tools* resource.

Learning Lines

Equations of Linear Relationships

Linear relationships are the continuous counterparts of arithmetic sequences. In previous units, students investigated and applied linear relationships and their properties. Now this knowledge is reviewed and formalized.

Different Representations of a Linear Relationship

Students read and draw graphs in a coordinate plane. Word formulas are used. Students convert equations to the formal notation using x and y as variables. There are several ways to draw a graph in a coordinate system:

- Make a table. Students should know that in order to draw a straight line, only two points are necessary.
- Start with the y-intercept and use the slope to plot another point.
- Use both the y-intercept and the x-intercept.

Concepts like *slope* and *y-intercept* were introduced and applied in *Graphing Equations*; however, the concept of the *x-intercept* is new. Students learn to relate the y-intercept and slope to the coefficients in the equation.

A geometric representation is used to illustrate the relationship between slope, y-intercept, and x-intercept. Equivalent equations are related to equivalent expressions.

Learning Outcomes

By the end of this section, students should be able to generalize linear relationships in different representations and recognize the concepts: slope, y-intercept, and x-intercept. Students are also able to draw a graph that corresponds to a linear equation.

Copyright © by Encyclopaedia Britannica, Inc. All rights reserved.

Algebra Rules!

Section C Operations with Graphs

Section Focus

In Section A, students added, subtracted, and multiplied sequences by a constant. These operations were carried out "number wise" and, more formally, as operations with expressions. In this section, the same operations are carried out with functions in a *continuous domain*. Likewise, there are different ways to carry out the operations graphically (that is, point by point) or algebraically. For the students, this connection demonstrates the *algebraic analogy*. The addition and subtraction of graphs is applied in other subjects like geography or economics. For example, depending on the economic problem, different types of costs need to be used separately as well as added or subtracted. Extra practice with similar problems can be found in the *Algebra Tools* resource.

In this section, other types of graphs are also used: The *line graph,* as used in statistics, is discussed in the first part of this section. Only the dots in the line graph are meaningful; the straight lines drawn between the dots are meant to show a "trend." The zigzag graph is an example of a *piecewise linear graph*.

Learning Lines

Equations of Linear Relationships

When adding or subtracting graphs, students learn to find new formulas for the sum or difference. This connects to Section A where expressions were added or subtracted. When adding graphs using a graphing calculator, the different meaning for y in the different graphs is made necessarily explicit: The graphing calculator uses y_1, y_2, and $y_3 = y_1 + y_2$. In this section, students also relate the addition of graphs to their symbolic counterpart. For example, given:

- graph A, with equation $y = 30 + 7x$ and
- graph B, with equation $y = 12 – 4x$,
- then the equation for graph A + B is $y = (30 + 7x) + (12 – 4x)$, which is also $y = 42 + 3x$.

Different Representations of a Linear Relationship

Graphs and equations are presented within and without a realistic context. The intersection point of two graphs is informally addressed.

Operations with Graphs

Graphs are added, subtracted, and multiplied by a constant, and the resulting equation is found. Connections are made to Section A where students added or subtracted expressions, and to Section B where students discussed the role of the slope and the x- and y-intercepts. The difference graph is used to find the intersection point of two graphs.

Number Sense

Working on the "walkway problem" provides an opportunity for students to work with fractions in a situation where you really need to simplify the formula.

Copyright © by Encyclopaedia Britannica, Inc. All rights reserved.

Algebra Rules!

Section C Operations with Graphs

Learning Outcomes

Students recognize how different formulas they saw in previous units are all part of the same "family" of linear relationships. They can use a graphing calculator to draw graphs, when appropriate (optional). Students are able to add, subtract, and multiply graphs by a constant. Students can also find the resulting equation for straight lines, both by operating with the formal equations and by drawing the resulting graph.

Copyright © by Encyclopaedia Britannica, Inc. All rights reserved.

Algebra Rules!

Section D Equations to Solve

Section Focus

Methods to solve linear equations are reviewed. New methods are also introduced. The relationship between solving an equation and the intersection point of two straight lines is emphasized. In this section, some teachers may prefer to use graphing calculators, while other teachers may prefer that their students have extra practice drawing graphs by hand.

This section begins with having students explore using the "cover method" for solving equations in various forms, including equations with rational expressions and radicals. Students are encouraged to use the "cover" method to build understanding of relationships embedded within equations.

Learning Lines

Solving Linear Equations

The "cover" method is used to solve linear equations. This method cannot be used when the variable appears on both sides of the equation. A familiar way to solve these types of equations, the "frog jumping problems" from the unit *Graphing Equations,* is reviewed. Students are then introduced to another method for solving equations, the "difference-is-0" method. This strategy can be used with a much larger range of equations than the cover method, and number strips are used as an introduction to this approach. Students choose the method they find most useful.

Equations of Linear Relationships

Solving equations is also related to intersecting lines. Students use the slope to determine whether two straight lines have an intersection point or not and relate the answer to an algebraic solution.

A linear equation with no solution is related to a pair of parallel lines. In the last question of the section, students discover that the difference graph of two parallel straight lines is a horizontal line.

Learning Outcomes

Students are able to solve linear equations in different forms using a variety of strategies. They relate solving equations to finding an intersection point of two straight lines, within a context as well as without a context.

Copyright © by Encyclopaedia Britannica, Inc. All rights reserved.

Algebra Rules!

Section E Operating with Lengths and Areas

Section Focus

In this section, we focus on students' flexibility in working with linear expressions and simple quadratic expressions. Equivalent forms are addressed. The mathematical content of earlier sections is reviewed and expanded. Students are only informally introduced to working with quadratic expressions like:

$$3p^2 + p^2 = 4p^2$$

$$(a+b)^2 = a^2 + 2ab + b^2$$

The familiar mathematical contexts of perimeter and area are chosen to work with expressions. Extra practice and extensions can be found in the *Algebra Tools* resource. Many examples of *geometric algebra* are provided. This idea of a geometrical underpinning of the algebra is quite old: Euclid's *Elements,* written over 2,000 years ago, provides many well-known examples of this mathematical relationship. The distributive property, reviewed within a different model, is made explicit and generalized.

Learning Lines

Linear Expressions

Expressions with more than one variable are found and investigated within the context of perimeter of two-dimensional shapes.

Quadratic Expressions

The context of area is used to introduce quadratic expressions. Students design a shape if an expression for its area and perimeter is given. Students investigate the algebraic rule $(a + b)^2 = a^2 + 2ab + b^2$ in a geometric model.

Equivalent Expressions

A rectangle model is used to show how equivalent expressions, including quadratic expressions, are represented by equal areas.

Learning Outcomes

Students are introduced, informally and pre-formally, to quadratic expressions. They are able to use the distributive property to represent algebraic expressions.

Copyright © by Encyclopaedia Britannica, Inc. All rights reserved.